PIRE Webinar: Introduction to Interferometry

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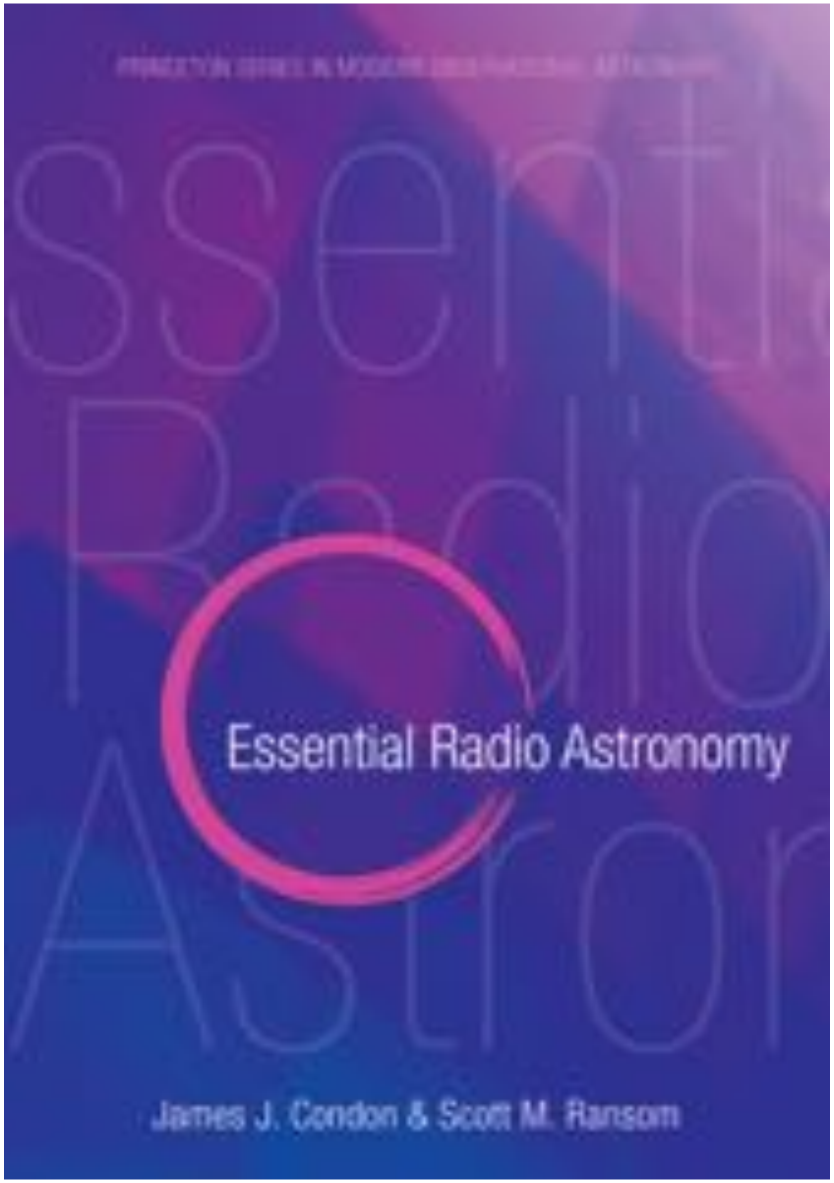
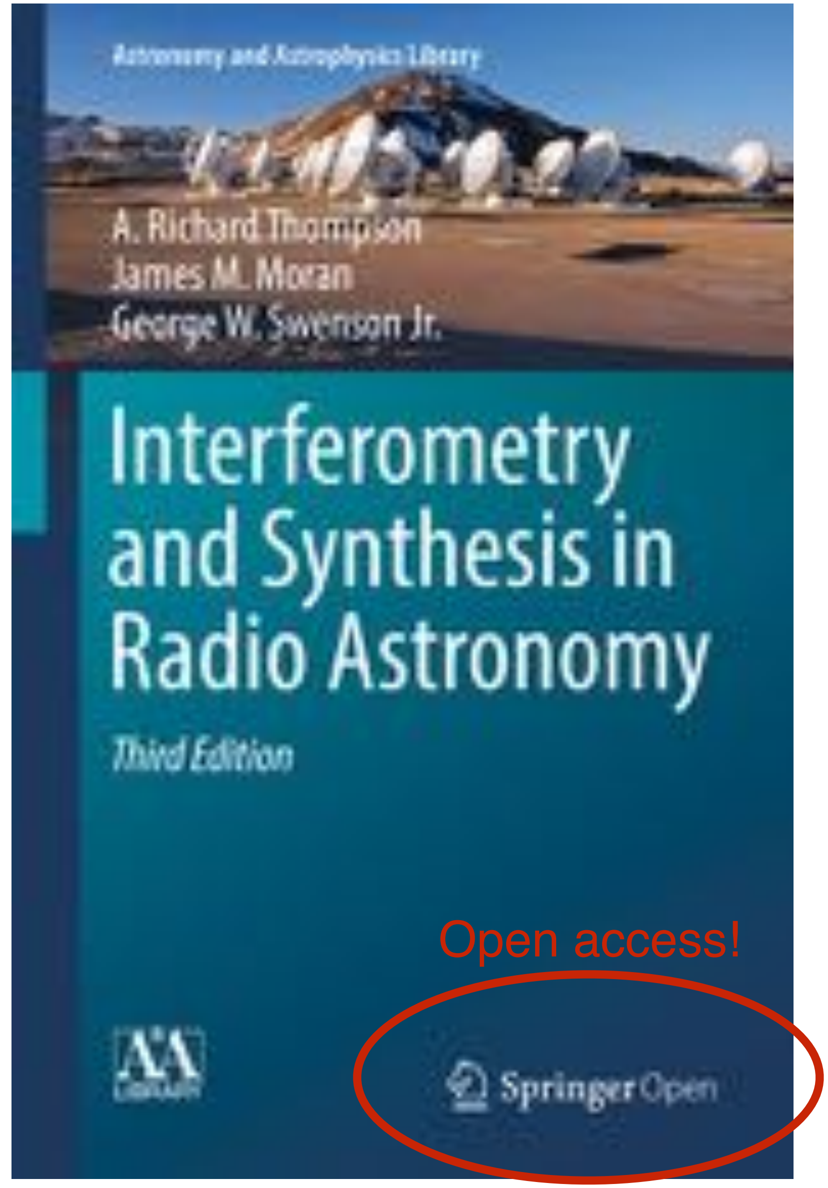
University of Arizona

Useful Resources



ERA



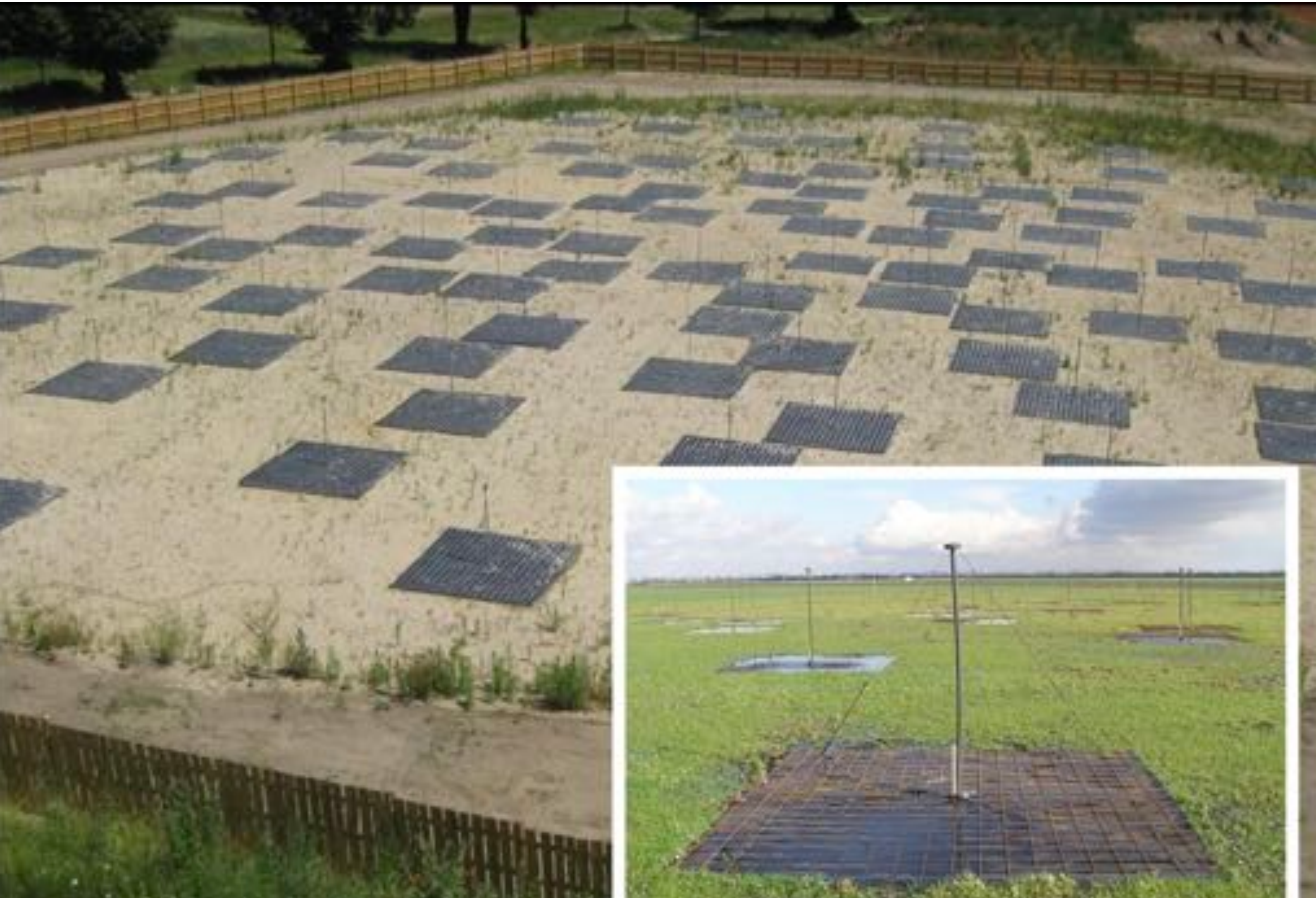
TMS

(Referenced in subsequent slides)

Radio Interferometers




Radio Interferometers



Why Interferometry?

- Resolution and collecting area

Telescope size, surface accuracy, and pointing are jointly limited

$$\theta_{res} \approx 2'' \times \frac{\lambda_{\rm cm}}{D_{\rm km}}$$

Limited?

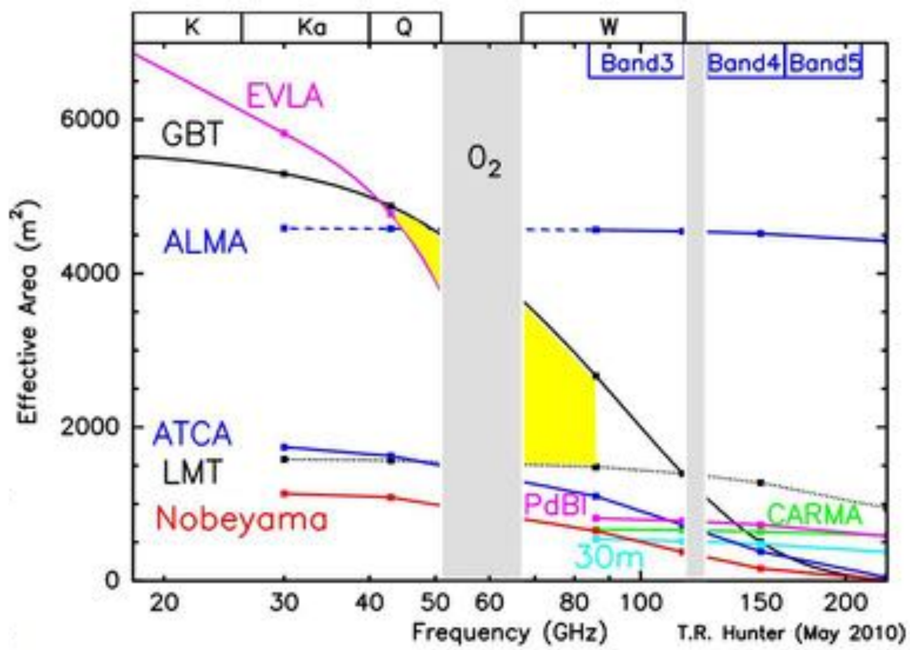
Limited!

Why Interferometry?

- Resolution and collecting area

  Telescope size, surface accuracy, and pointing are jointly limited

- Interferometers can provide:
  - highest resolution (EHT: λ/D >10Gλ!)
  - largest collecting area →
  - large number of resolution elements
    large field of view with high sensitivity
  - highest astrometric precision



Antenna Illumination and Beam

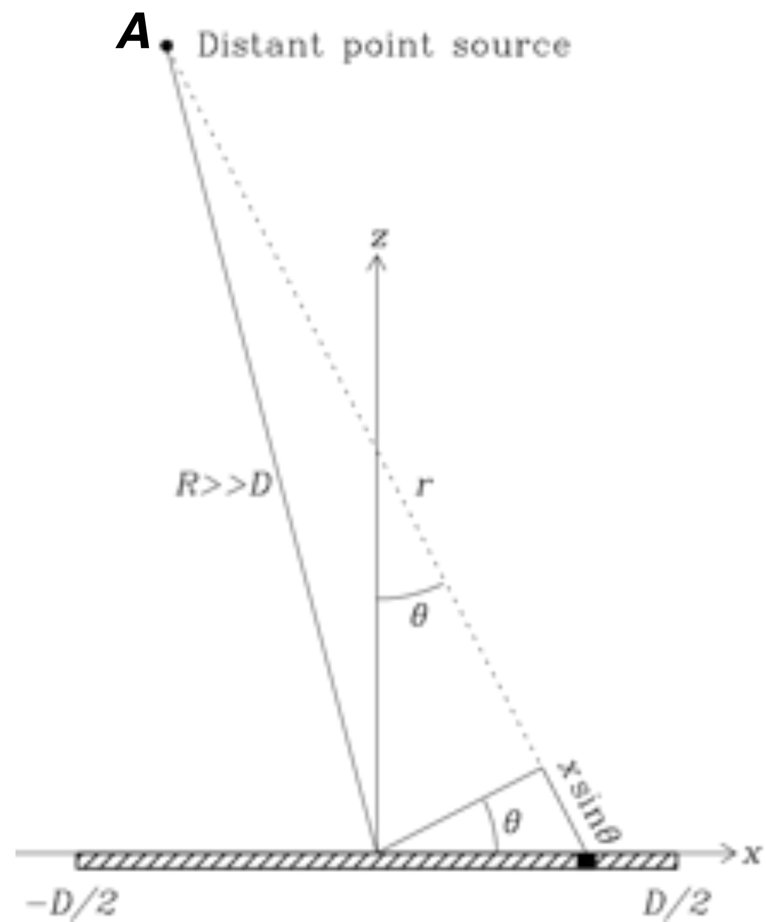
Consider a 1-D antenna of length D transmitting at frequency ν ($\lambda = c/\nu$)

Calculate field at point **A** at large distance R

Consider small segment dx at position x, with field $g(x)$.

Electric field contribution at point **A** is

$$dE \propto \frac{g(x)}{r(x)} \exp(-i2\pi r(x)/\lambda)\, dx$$



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In coefficient: $1/r(x) \simeq 1/R$

In exponent: $r(x) = R + x sin\,\theta \approx R + xl$ (for small θ, and $l \equiv sin\,\theta$)

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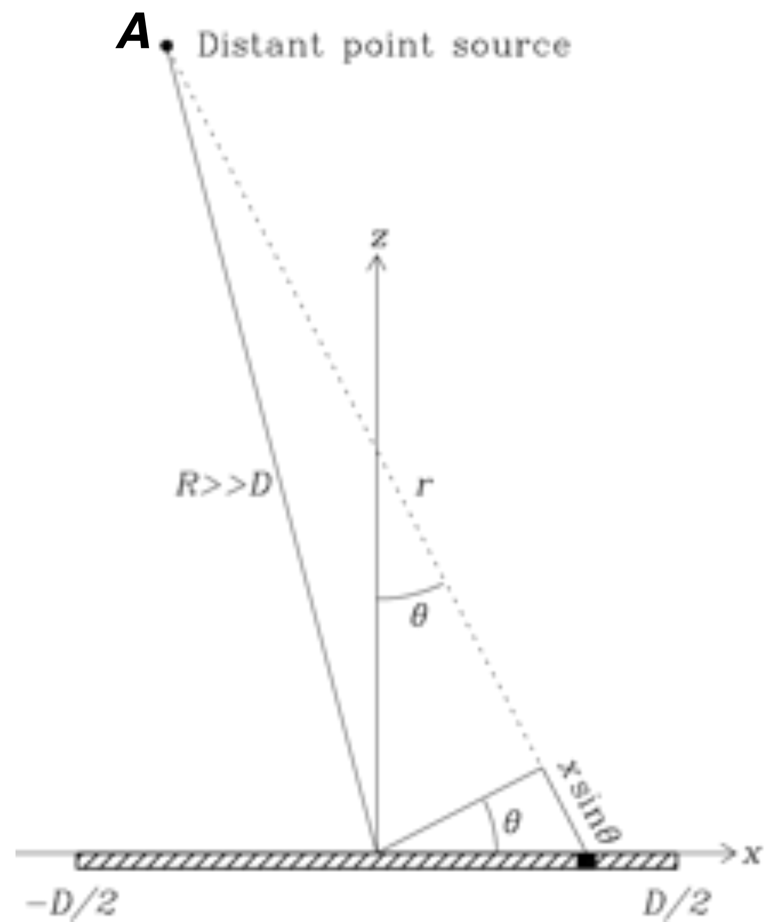

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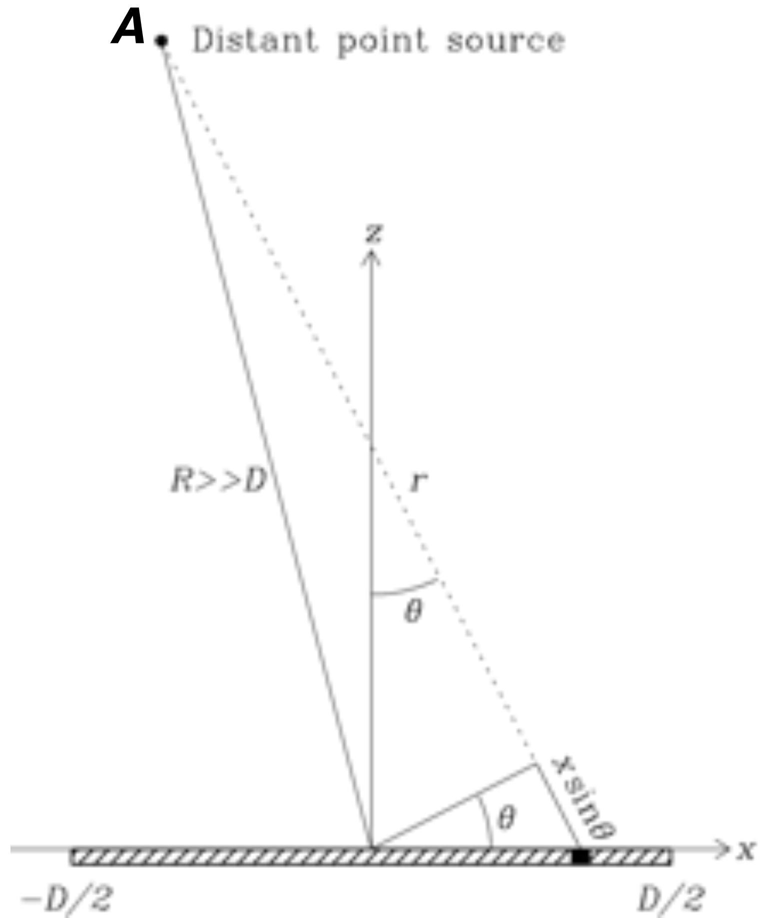
$$dE \propto \frac{g(x)}{R} \exp(-i2\pi R/\lambda) \exp(-i2\pi xl/\lambda)\, dx$$

Now, define $u \equiv x/\lambda$, absorb constants into g, and integrate to get $E(\boldsymbol{A})$

$$E = \int g(u)\, e^{-i2\pi ul} du$$



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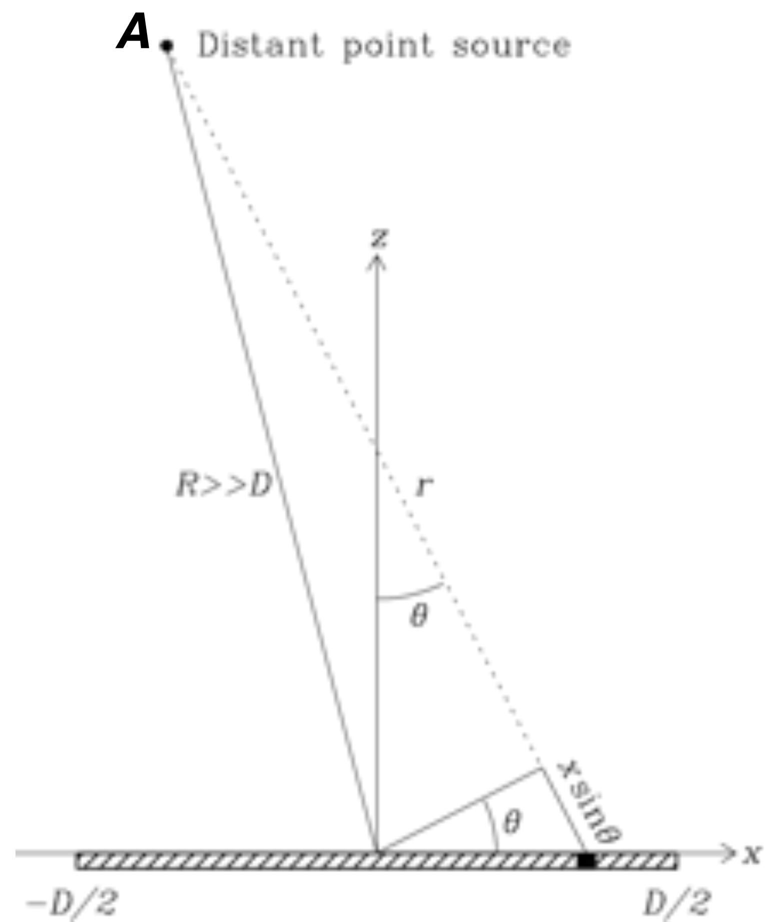
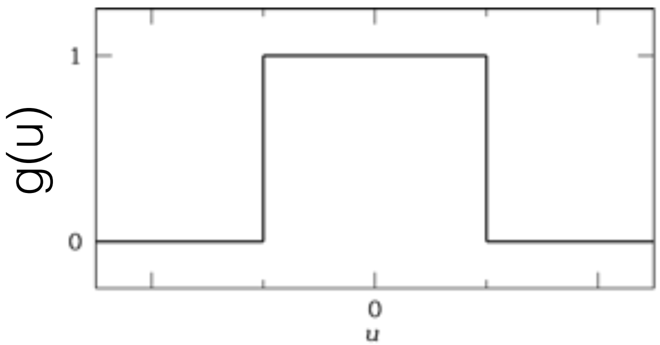


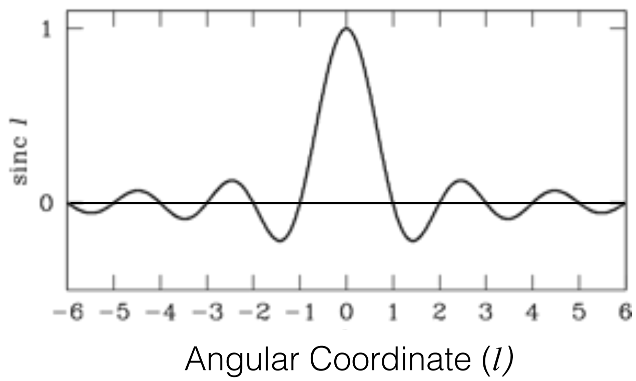
- Antenna beam is **Fourier Transform** of antenna illumination pattern *g(x)*!

Antenna Illumination and Beam

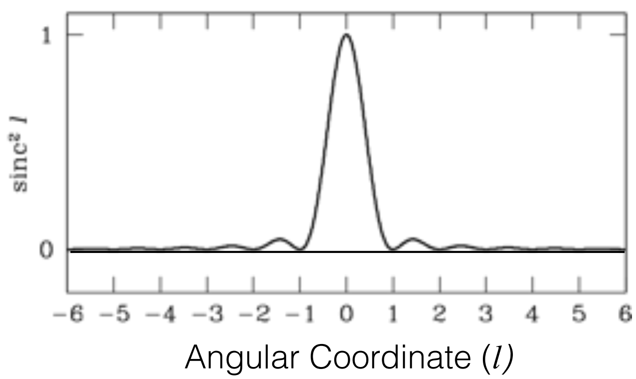
Antenna Illumination Pattern



Field Pattern



Power Pattern (PSF)

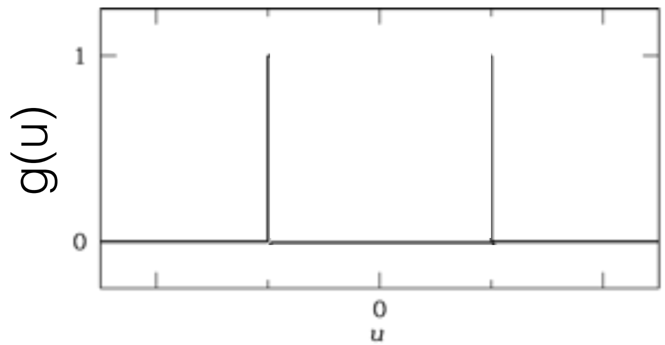


- Larger aperture in λ/D
  → smaller beam on sky
  (FT similarity theorem)
- Actual antennas usually “taper” g(u)
- Blockage, surface errors, etc., can be included in beam pattern via FT

Antenna Illumination and Beam

- Two small apertures -> plane wave

Antenna Illumination Pattern



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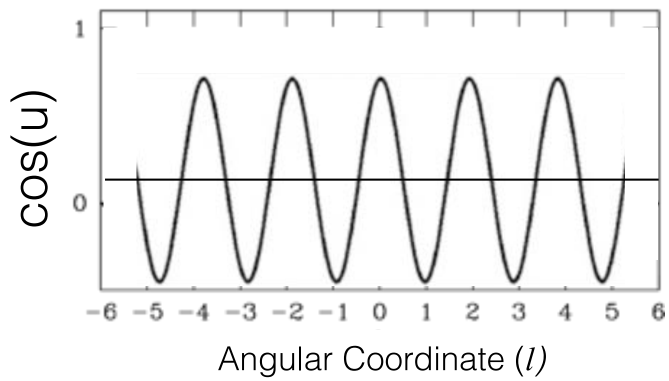

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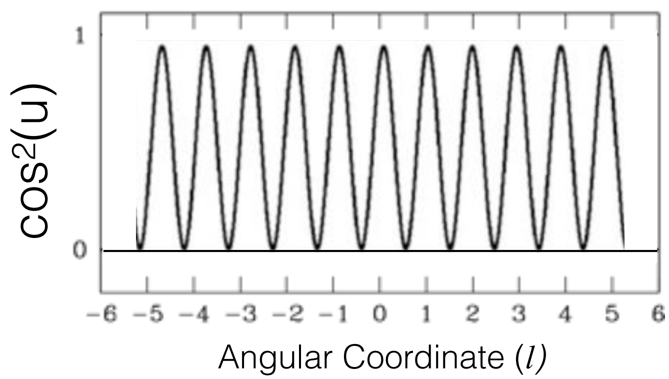
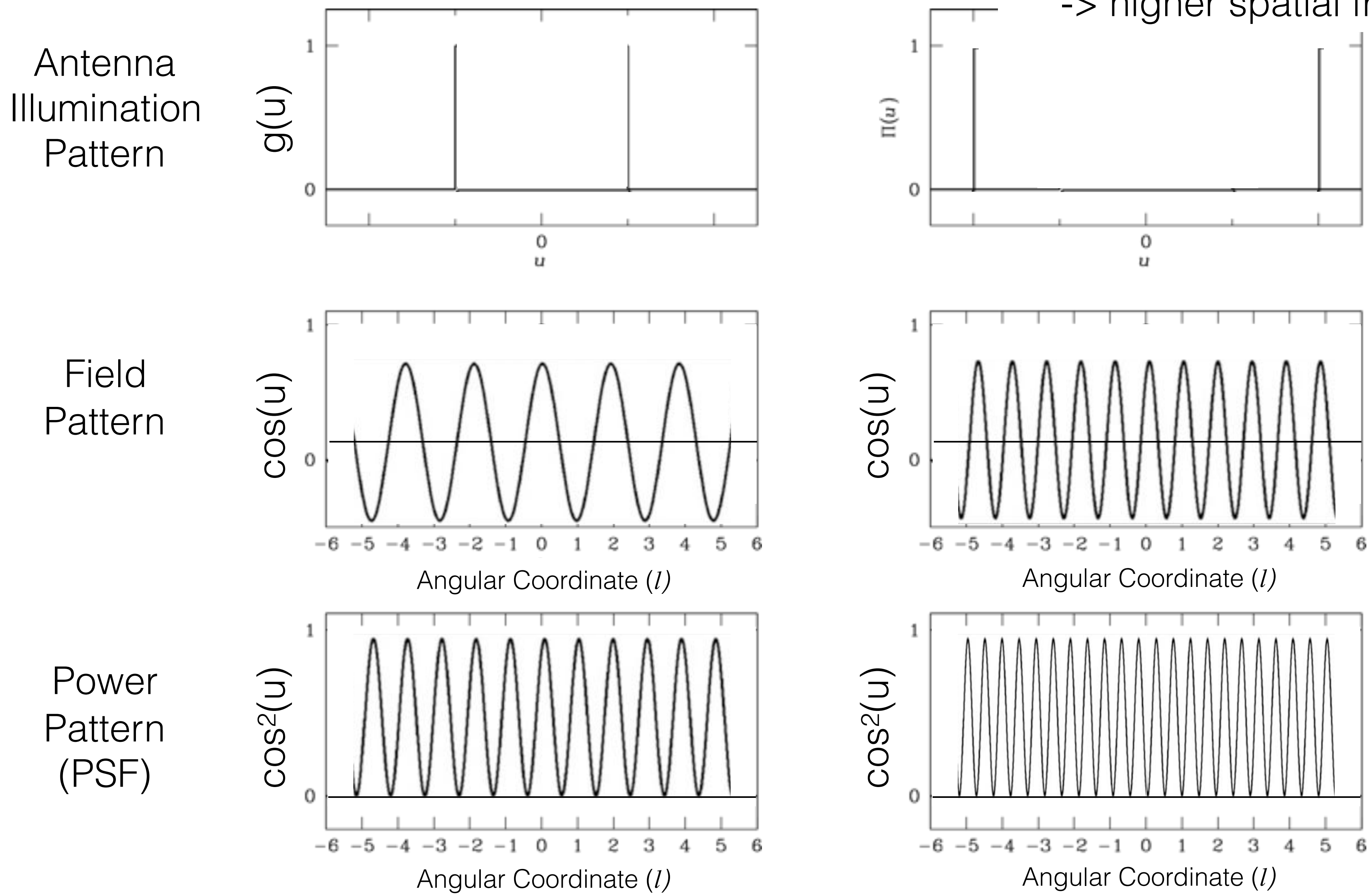


Antenna Illumination and Beam

- Two small apertures -> plane wave
- Wider separation
  -> higher spatial frequency

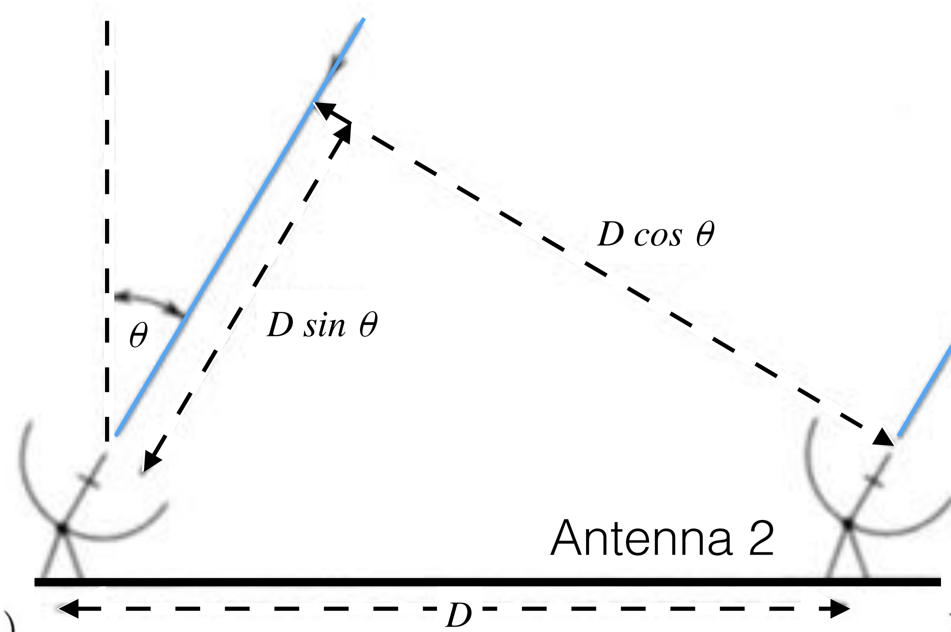


TMS 2.1,2.2

Two-Element Analysis

Assumptions:

- Distant point source
  (→ plane wave)
- Monochromatic



$$V_1 = V\, cos\, 2\pi\nu(t - \tau_g)$$

$$\tau_g = \frac{D\, sin\, \theta}{c}$$

$$V_2 = V\, cos\, 2\pi\nu t$$

Two-Element Analysis

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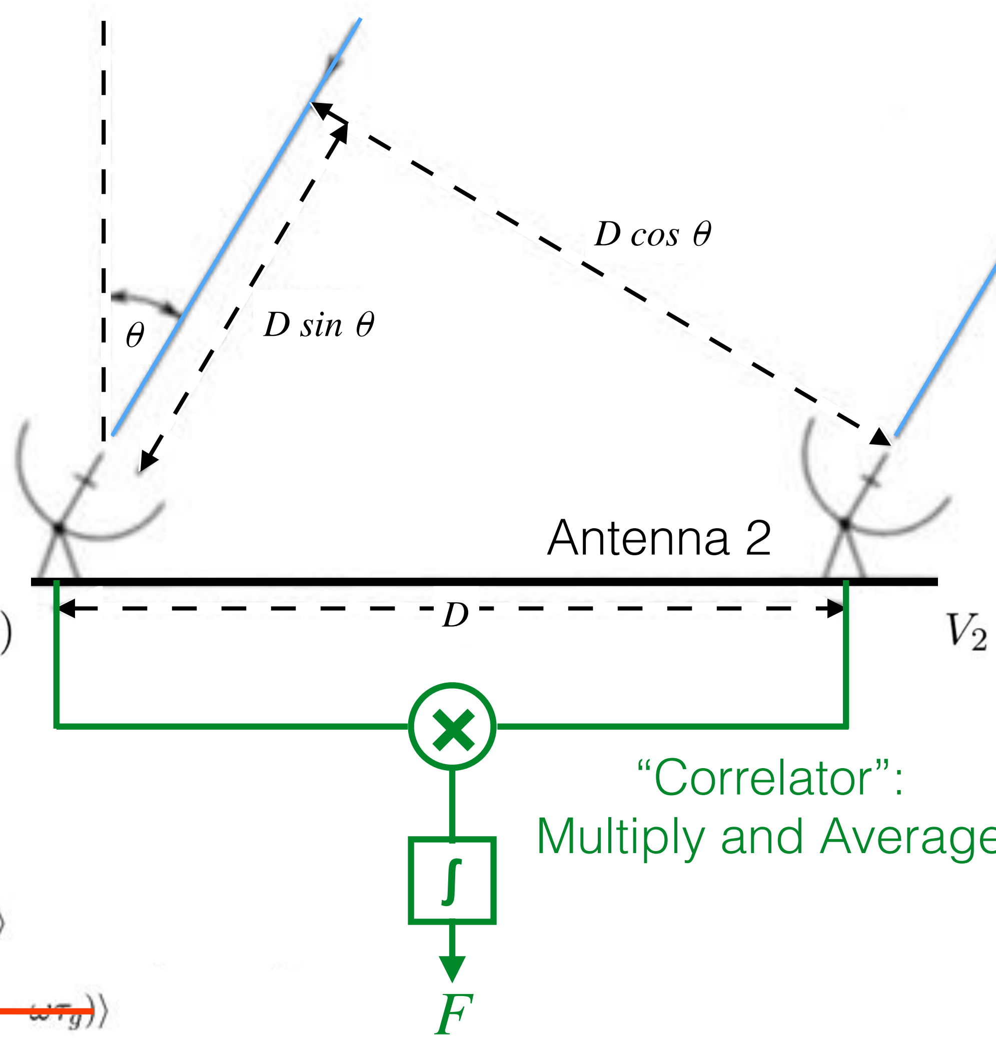

$V_1 = V\,cos\,2\pi\nu(t - \tau_g)$

$V_2 = V\,cos\,2\pi\nu t$

$$\tau_g = \frac{D\,sin\,\theta}{c}$$

$$F = \langle V_1 V_2 \rangle = \langle V^2 cos\,\omega t\,cos\,\omega(t - \tau_g)\rangle$$

$$= \frac{V^2}{2}\langle cos\,\omega\tau_g + \cancel{cos\,(2\omega t - \omega\tau_g)}\rangle$$

→0 when averaging over t >>1/(2ω)

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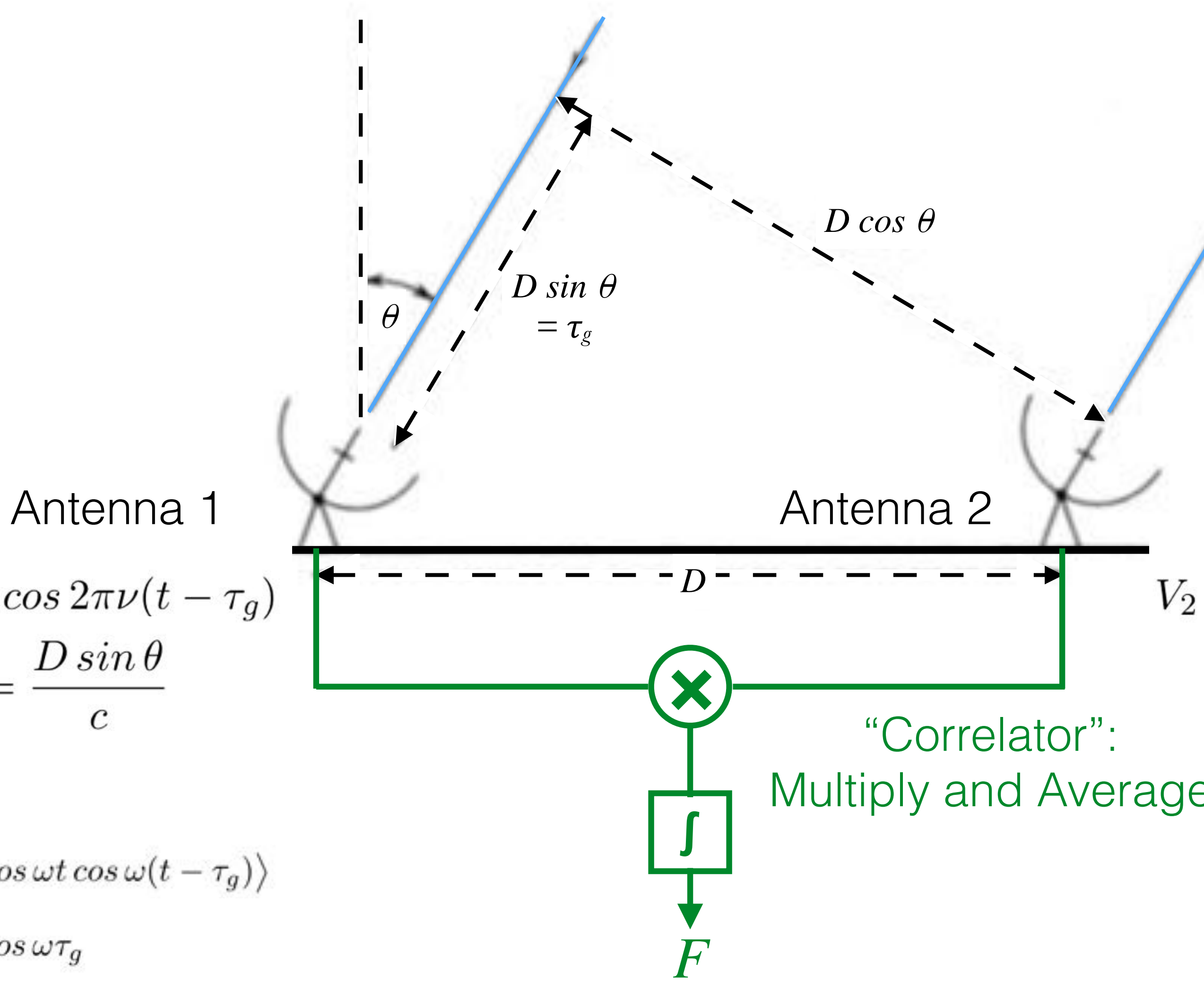

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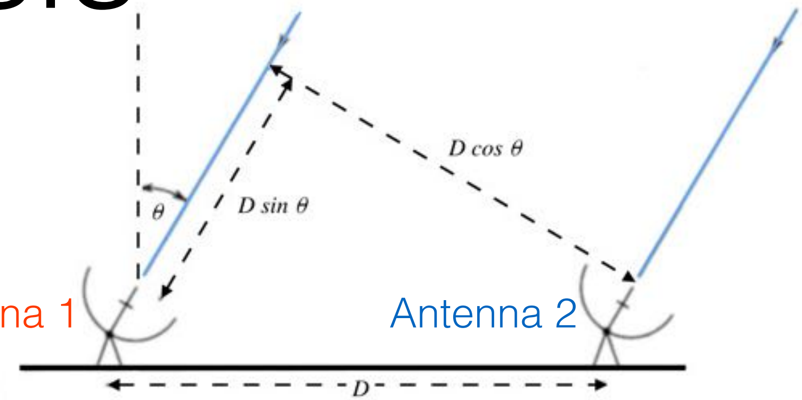
$$F = \langle V_1 V_2 \rangle = \langle V^2 cos\,\omega t\,cos\,\omega(t - \tau_g)\rangle$$

$$= \frac{V^2}{2} cos\,\omega\tau_g$$

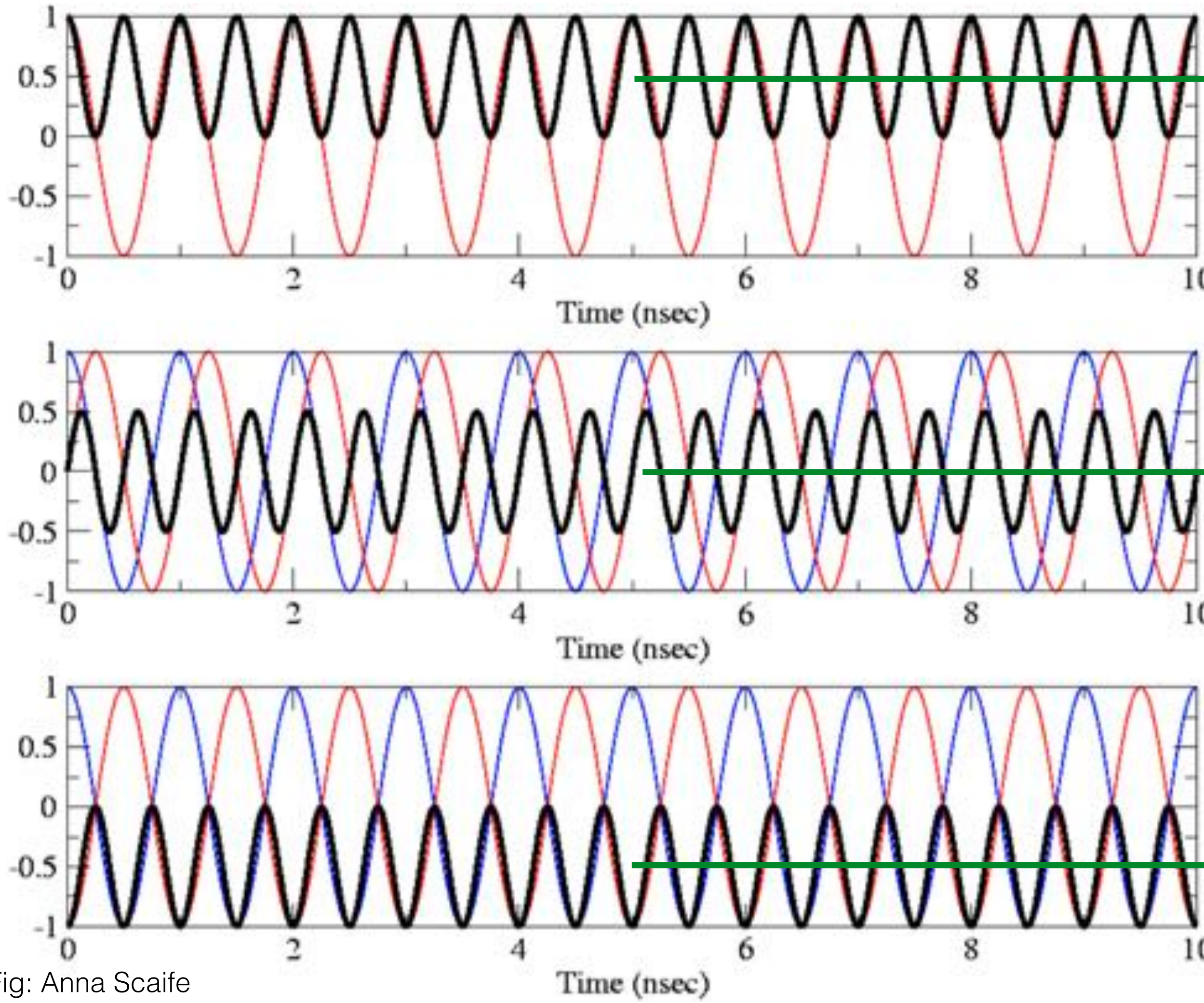
$$F = \frac{V^2}{2} cos\left(2\pi \frac{D}{\lambda} sin\,\theta\right)$$

Periodic oscillation, as D sin Θ changes by λ

Two-Element Analysis



Monochromatic 1GHz signal



$F = 0.5$

A/B in phase

$F = 0$

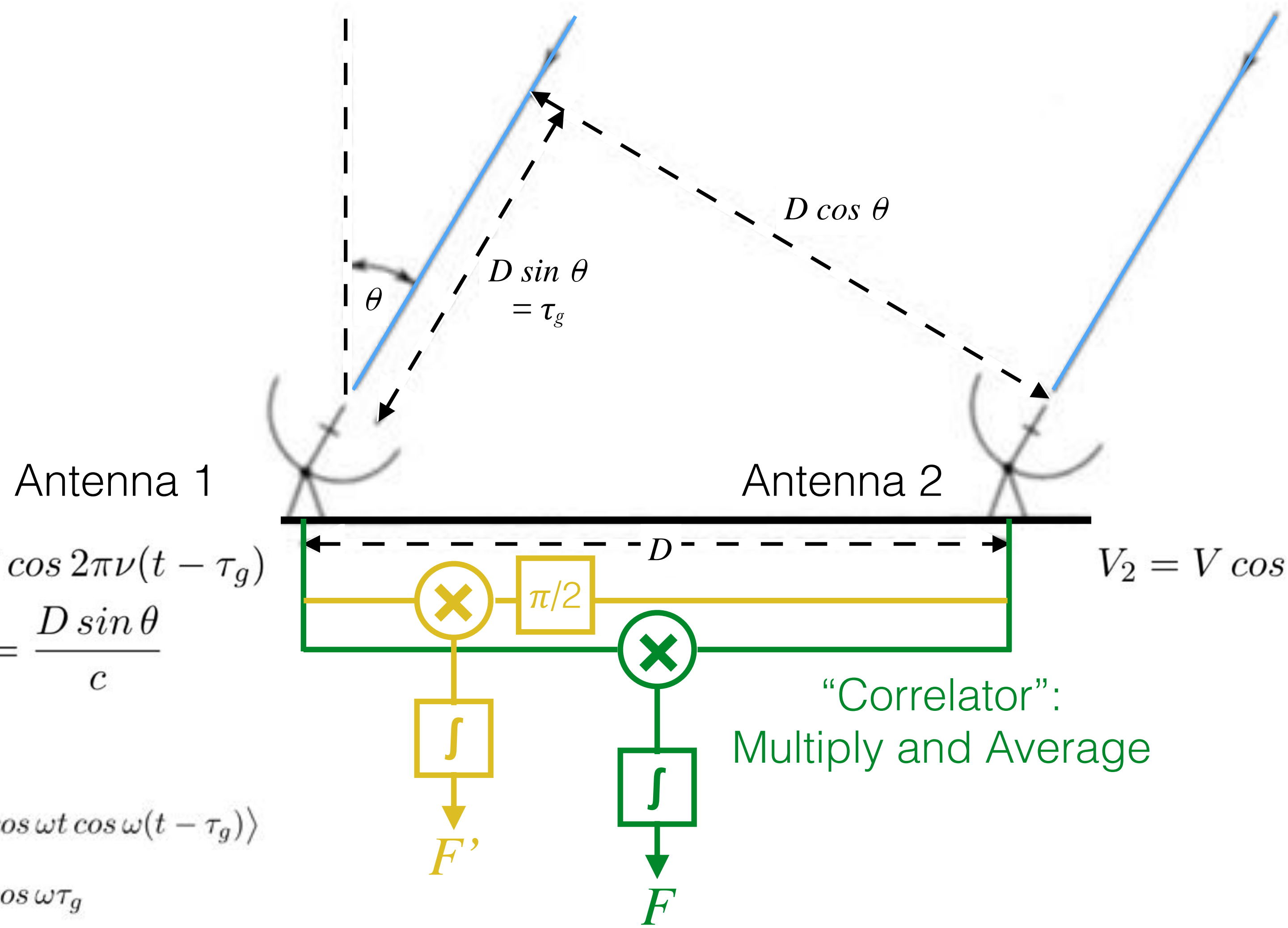
A/B in quadrature phase
(D sin Θ = 1,3,5…λ/4)

A/B in anti-phase

$F = -0.5$

Fig: Anna Scaife

Two-Element Analysis



$$V_1 = V\,cos\,2\pi\nu(t - \tau_g)$$

$$V_2 = V\,cos\,2\pi\nu t$$

$$\tau_g = \frac{D\,sin\,\theta}{c}$$

$$F = \langle V_1 V_2 \rangle = \langle V^2 cos\,\omega t\,cos\,\omega(t - \tau_g)\rangle$$

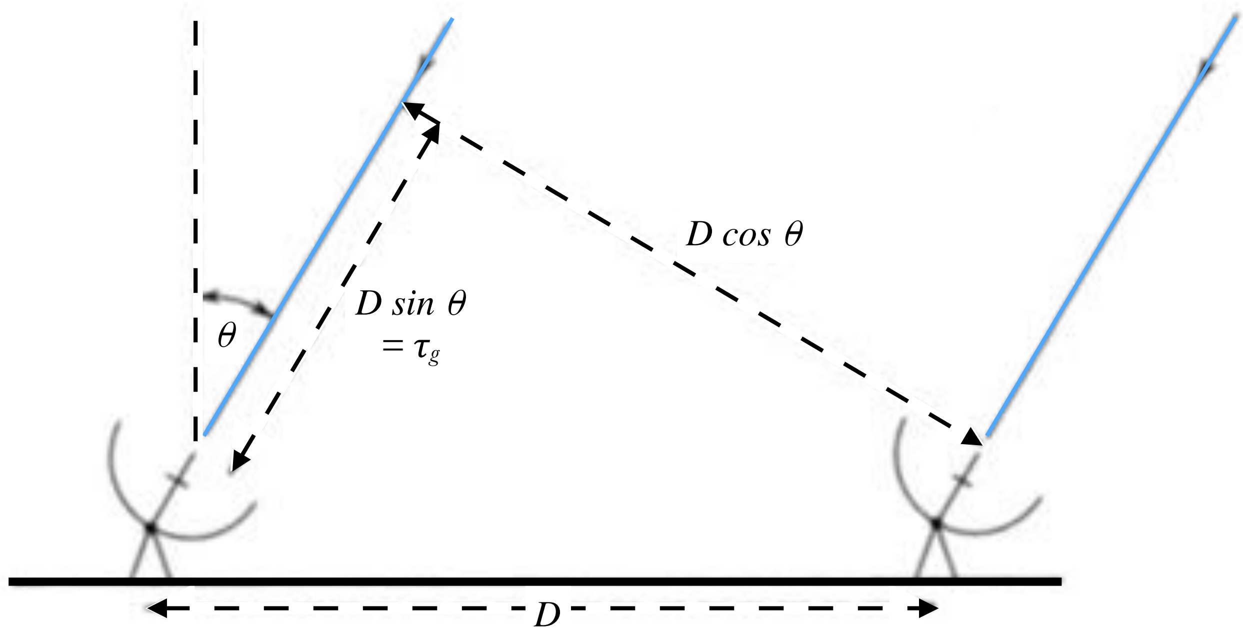
$$= \frac{V^2}{2} cos\,\omega\tau_g$$

$$F = \frac{V^2}{2} cos\left(2\pi\frac{D}{\lambda} sin\,\theta\right)$$

Complex correlator:
F' provides missing fringe information

TMS 2.1,2.2

Two-Element Analysis



Resolution?

$$\Delta\left(2\pi\frac{D}{\lambda}sin\,\theta\right) \sim 2\pi$$

$$\frac{D}{\lambda}\Delta\left(sin\,\theta\right) \sim 1$$

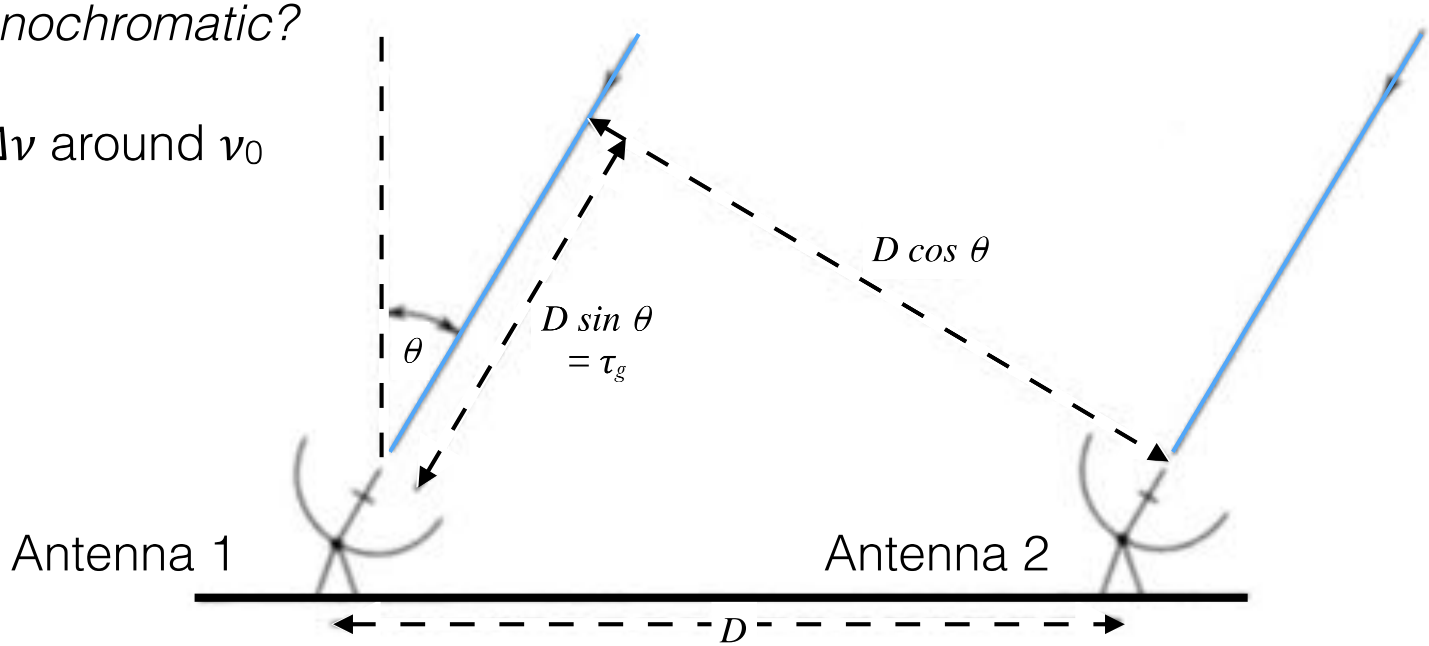
$$\boxed{\Delta\theta \sim \frac{\lambda}{D}}$$

"Fringe spacing" is determined by telescope separation

Two-Element Analysis

What if signal is not monochromatic?

Assume bandwidth $\Delta\nu$ around ν_0

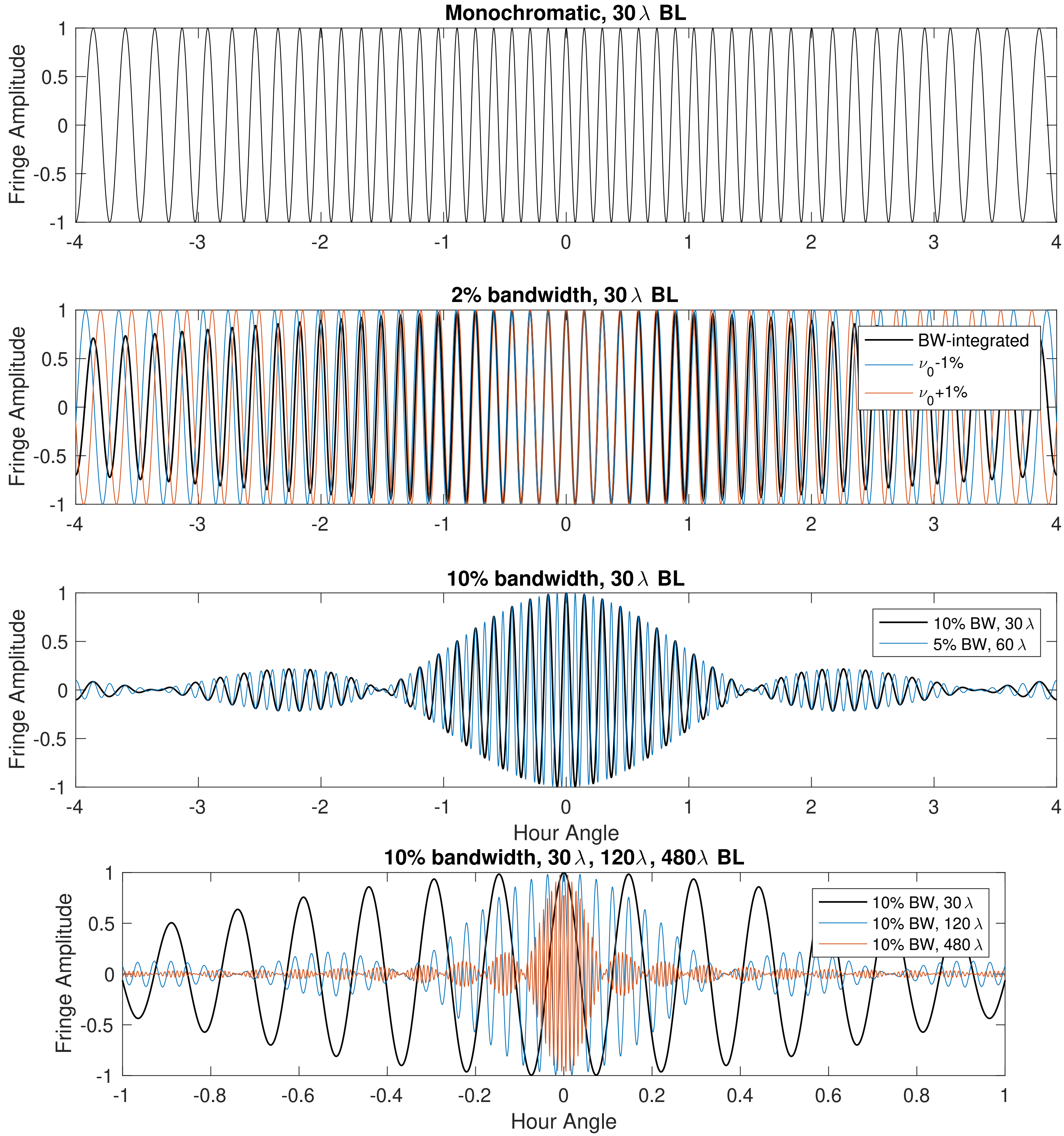


$$F = \frac{1}{\Delta\nu} \int_{\nu_0-\Delta\nu/2}^{\nu_0+\Delta\nu/2} \frac{V^2}{2} \cos\left[\frac{2\pi D\nu}{c} \sin\theta\right] d\nu$$

$$\begin{aligned} F &= \frac{V^2}{2} \cos\left[\frac{2\pi D\nu_0}{c} \sin\theta\right] \frac{\sin\left[\frac{\pi D\Delta\nu}{c}\sin\theta\right]}{\frac{\pi D\Delta\nu}{c}\sin\theta} \\ &= F_{\nu_0}\, \mathrm{sinc}\left[\frac{D\Delta\nu}{c}\sin\theta\right] = F_{\nu_0}\, \mathrm{sinc}\left[\tau_g \Delta\nu\right] \end{aligned}$$

$$\tau_g = \frac{D \sin\theta}{c}$$

Same fringe, with sinc (=sin(πx)/(πx)) envelope

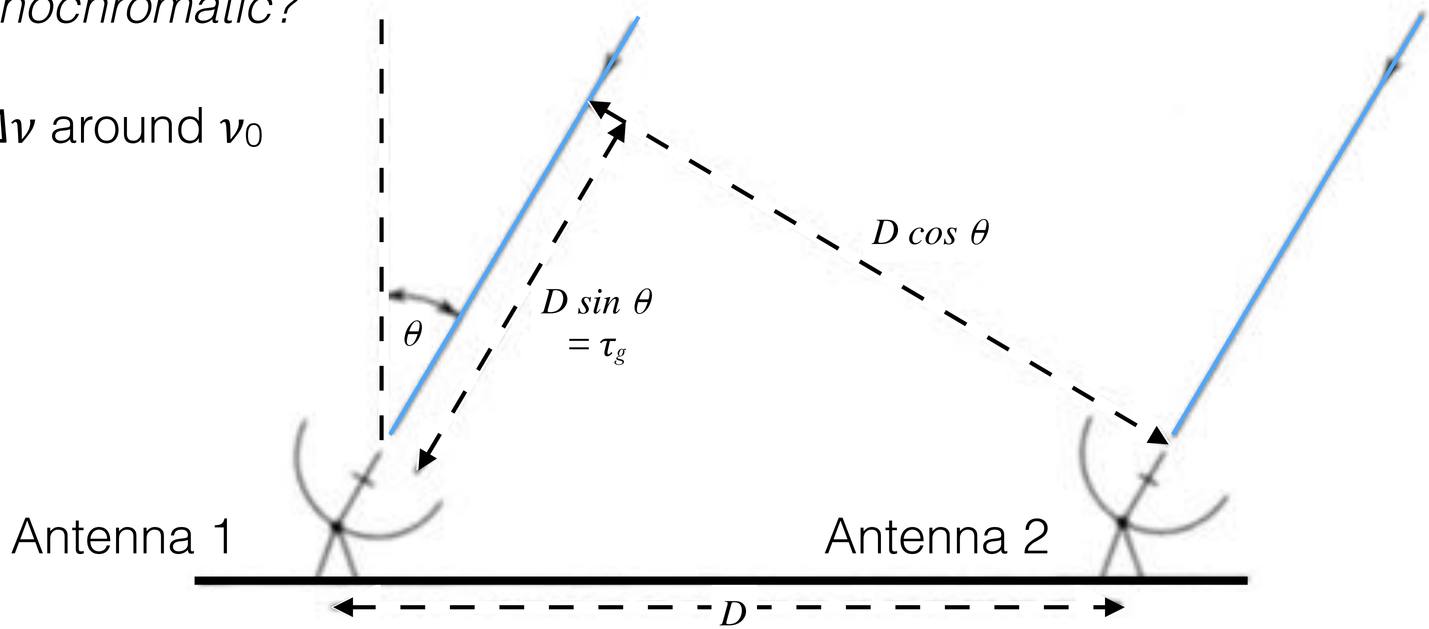

Monochromatic, 30 λ BL
Fringe Amplitude
2% bandwidth, 30 λ BL
BW-integrated
ν_0-1%
ν_0+1%
10% bandwidth, 30 λ BL
10% BW, 30 λ
5% BW, 60 λ
Hour Angle
10% bandwidth, 30 λ, 120λ, 480λ BL
10% BW, 30 λ
10% BW, 120 λ
10% BW, 480 λ
Hour Angle

Two-Element Analysis

TMS 2.1,2.2

What if signal is not monochromatic?

Assume bandwidth $\Delta\nu$ around ν_0



$$F = F_{\nu_0}\, sinc\,[\tau_g \Delta\nu]$$

Bandwidth restriction is extremely stringent for useful baseline lengths

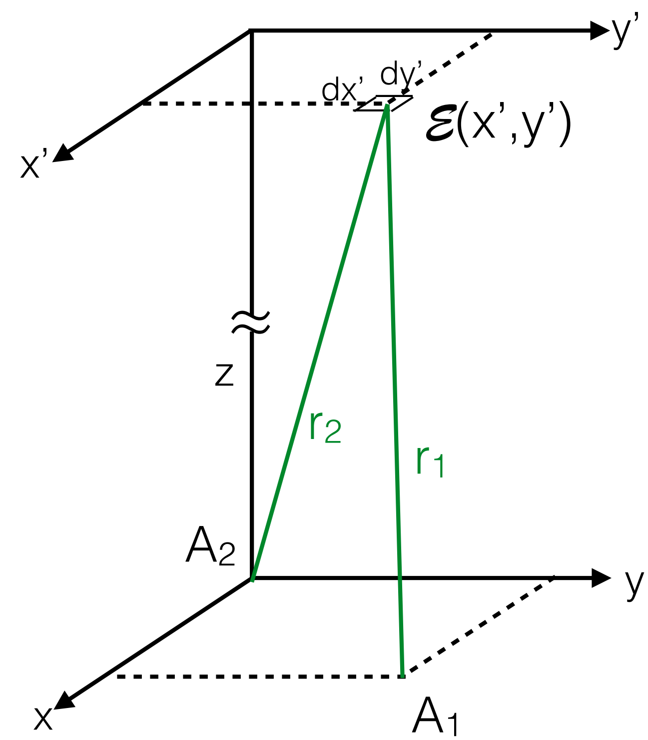
$\Delta\nu \ll 1/\tau_g$ 1km baseline implies $\Delta\nu \ll$ 300kHz

Interferometers designed for imaging small scales must remove τ_g

This is "Delay tracking"

Interferometric Measurement

- Idealized interferometer
  - Two antennas, A_1 at (x,y) and A_2 at (0,0) for simplicity
  - Source in direction *r* emitting electric field $\mathcal{E}(x',y')$
- Simplifying assumptions (can be relaxed)
  - Source is very distant
  - Monochromatic source
  - Ignore polarization
  - Source is spatially incoherent
  - Nothing between antennas and source

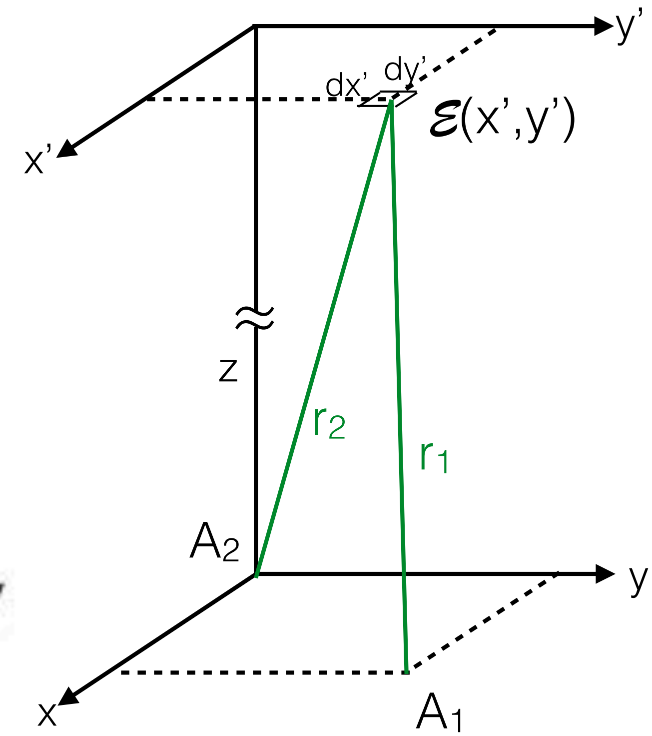


Interferometric Measurement

- What do we measure when we multiply E fields at two antennas?

- Field received at Antenna 1 (similar for A_2):

$$E_1(x', y', t) = \frac{\mathcal{E}(x', y', t - r_1/c)}{z} e^{-2\pi i\nu(t - r_1/c)}$$



- Define the spatial correlation function between fields measured at positions of A_1 and A_2

$$R_{12}(x', y', \vec{r_1}, \vec{r_2}) = \langle E_1 E_2^* \rangle$$
$$= \frac{1}{z^2} \langle \mathcal{E}(x', y', t)\mathcal{E}^*(x', y', t) \rangle \, e^{2\pi i\nu(\vec{r_1} - \vec{r_2})/c} dx' \, dy'$$

  - Note: this is time-averaged source intensity *I(x', y')*

$$\langle \mathcal{E}(x', y', t)\mathcal{E}^*(x', y', t) \rangle \equiv I(x', y')$$

* Assumed sampled bandwidth small compared to frequency so that field is similar across propagation time difference to drop r from $\mathcal{E}$

Interferometric Measurement

- What do we measure when we multiply E fields at two antennas?

- Simplify distance difference:
  - If source far away, z ≫ (x'-x), and so:

$$r_1 = \sqrt{z^2 + (x' - x_1)^2 + (y' - y_1)^2} \approx z\left[1 + \frac{(x' - x_1)^2}{2z^2} + \frac{(y' - y_1)^2}{2z^2}\right]$$

$$r_2 = \sqrt{z^2 + x'^2 + y'^2} \approx z\left[1 + \frac{x'^2}{2z^2} + \frac{y'^2}{2z^2}\right]$$

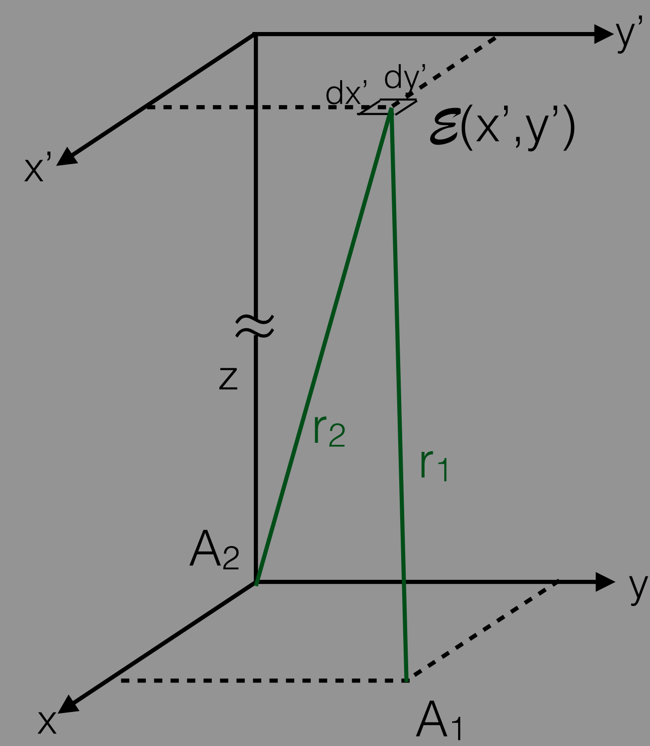
  - Difference between r_1 and r_2:

$$r_1 - r_2 \approx -\frac{1}{z}\left[x'D_x + y'D_y - \frac{1}{2}\left(D_x^2 + D_y^2\right)\right]$$

($D_x = x_1 - x_2$)

  - And if wavefront curvature small (in "far field"), then:

$$r_1 - r_2 \approx -\frac{x'D_x}{z} - \frac{y'D_y}{z}$$



Interferometric Measurement

- What do we measure when we multiply E fields at two antennas?

- Define convenient coordinates
  - Convert x', y' to angles:

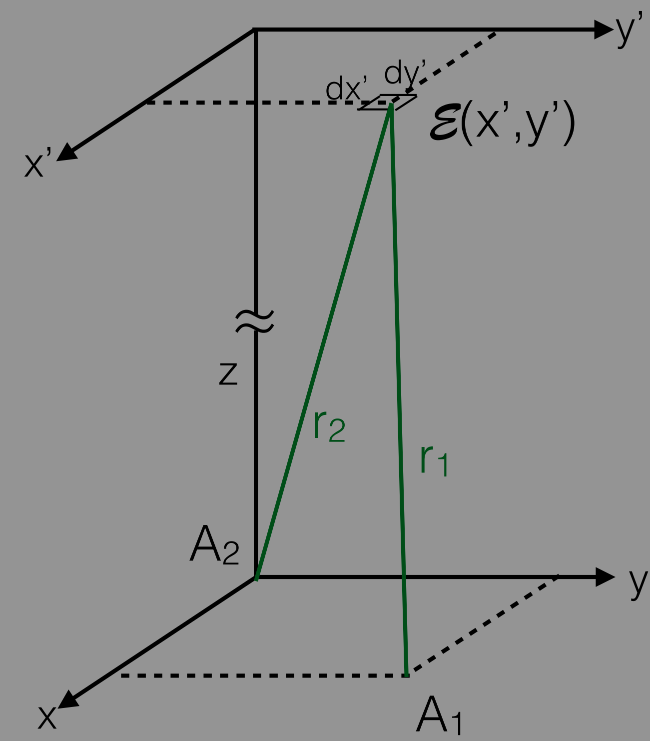
$$l = \frac{x'}{z} \qquad m = \frac{y'}{z} \qquad \qquad \frac{dx'}{z} = dl \qquad \frac{dy'}{z} = dm$$

  - Express D_x , D_y in wavelengths

$$u = \frac{D_x}{\lambda} \qquad v = \frac{D_y}{\lambda}$$

  - Simplify:

$$\frac{\nu}{c}(r_1 - r_2) = \frac{r_1 - r_2}{\lambda} \approx -(ul + vm)$$



TMS 15.1

Interferometric Measurement

- What do we measure when we multiply E fields at two antennas?

- Put it all back together:

$$R_{12}(x', y', \vec{r_1}, \vec{r_2}) = \langle E_1 E_2^* \rangle$$

$$= \frac{1}{z^2} \langle \mathcal{E}(x', y', t) \mathcal{E}^*(x', y', t) \rangle \; e^{2\pi i \nu (\vec{r_1} - \vec{r_2})/c} dx' \, dy'$$

$$-(ul + vm)$$

$$I(x', y')$$

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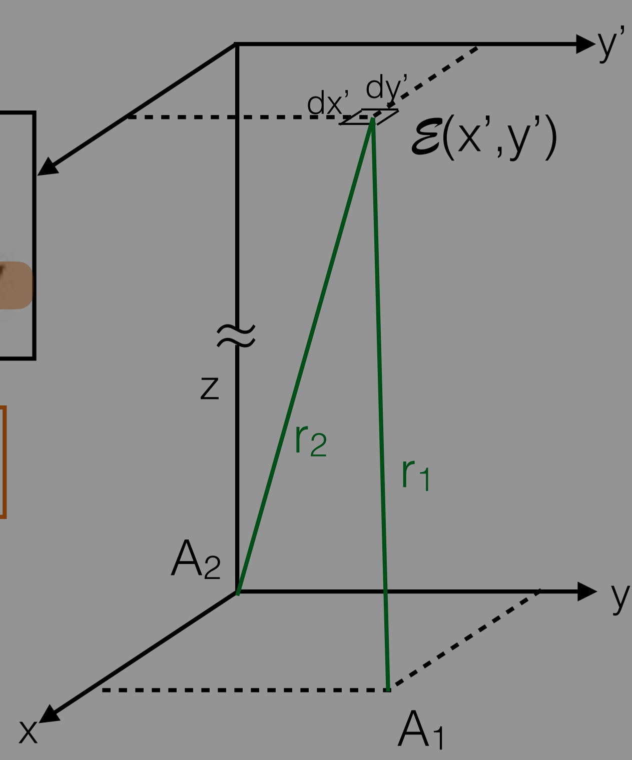

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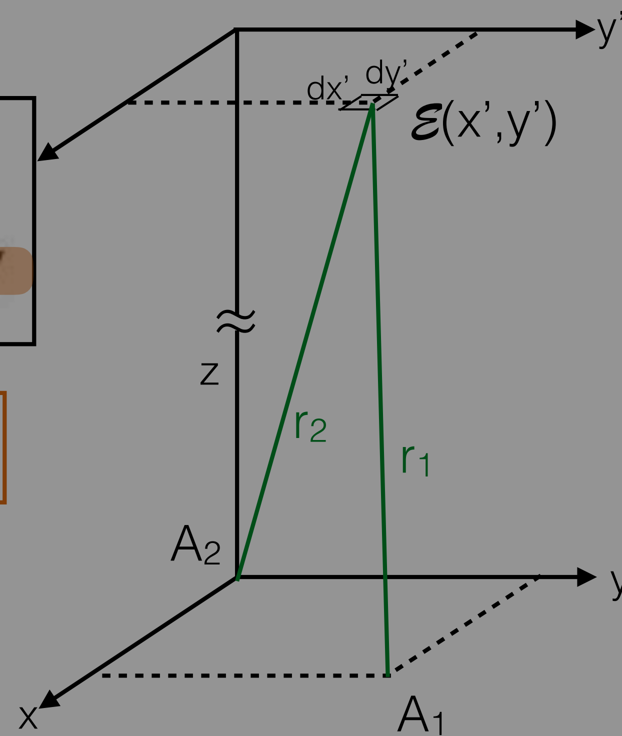
$-(ul + vm)$

$I(x', y')$

$l = \frac{x'}{z} \qquad m = \frac{y'}{z}$

$\frac{dx'}{z} = dl \qquad \frac{dy'}{z} = dm$



- This becomes:

$$R_{12}(x', y') = I(l, m)\, e^{-2\pi i\,(ul+vm)} dl \, dm$$

Interferometric Measurement

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$$I(x', y')$$

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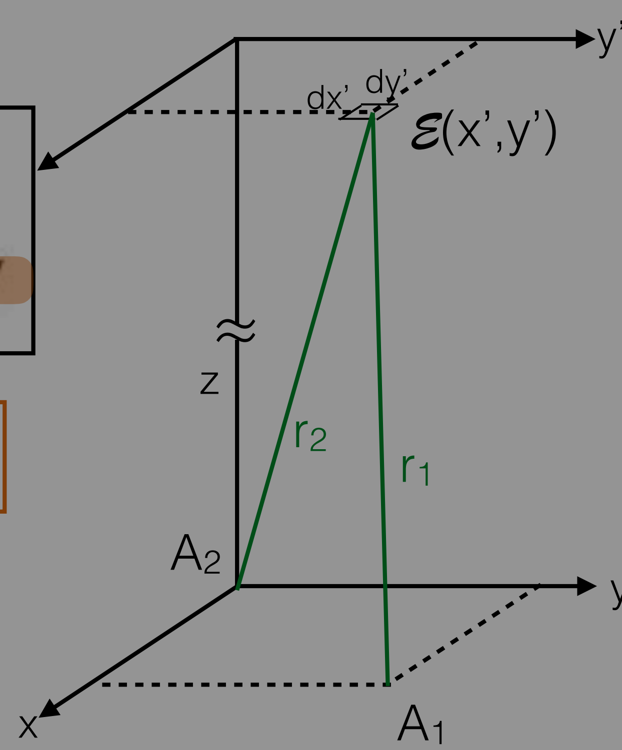


- This becomes:

$$R_{12}(x', y') = I(l, m) \, e^{-2\pi i \, (ul + vm)} dl \, dm$$

- Now, define "**visibility**" as integral of R_{12} over sky:

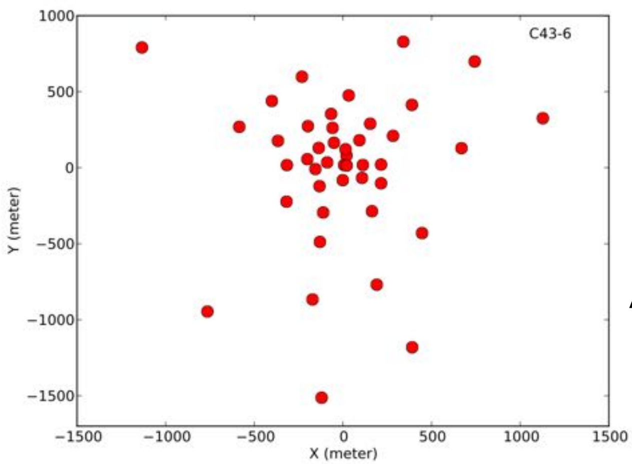
$$V(u, v) = \iint I(l, m) \, e^{-2\pi i \, (ul + vm)} dl \, dm$$

It's a Fourier Transform!
One spatial frequency
(u,v) measured per baseline

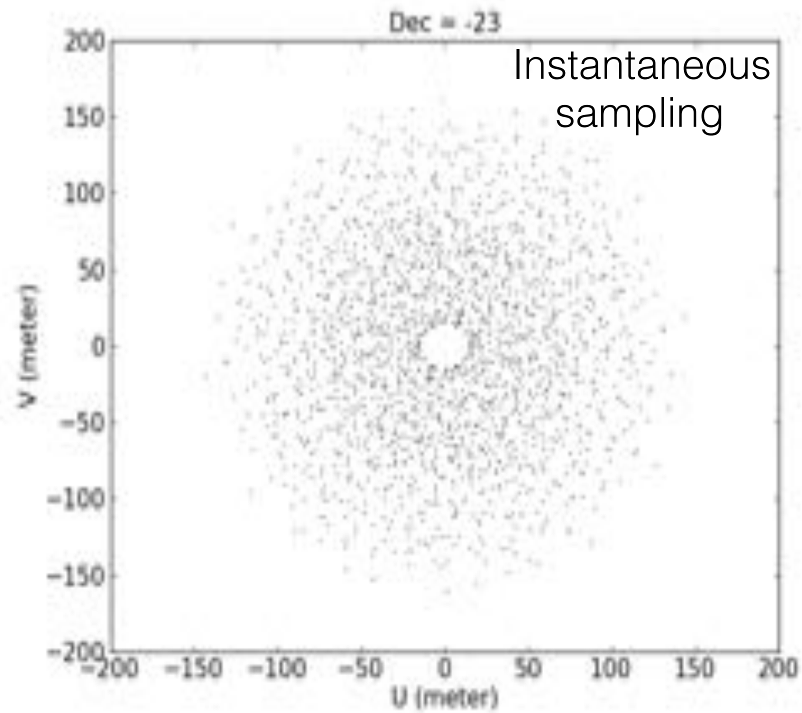
Fourier Sampling

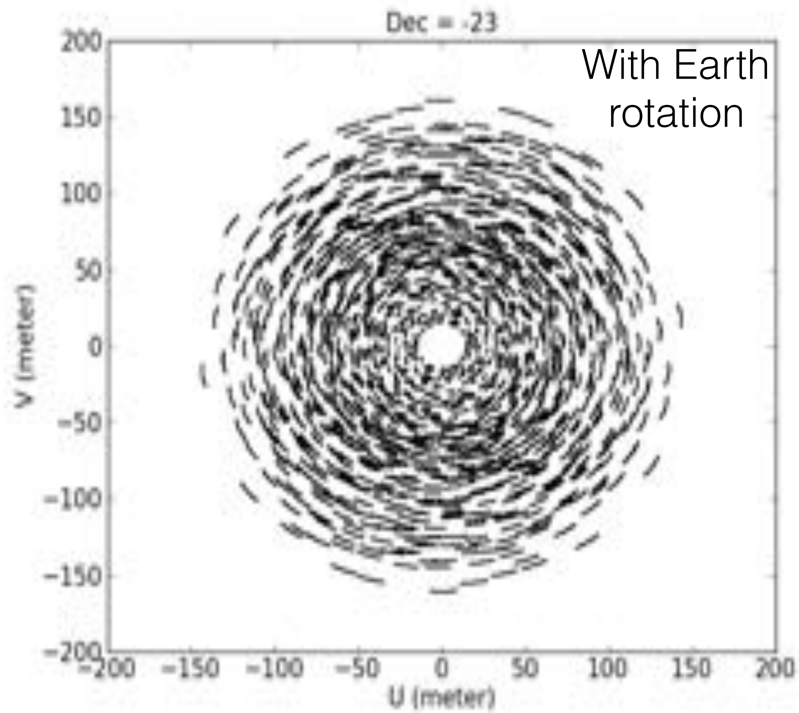
- Of course, interferometers only sample some *uv* spacings

Example: ALMA



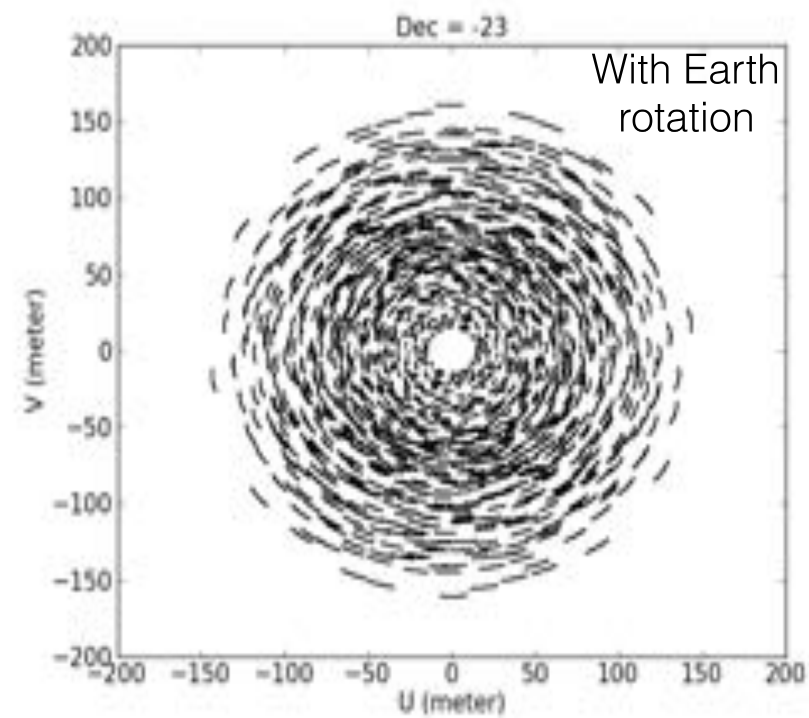
Array configuration





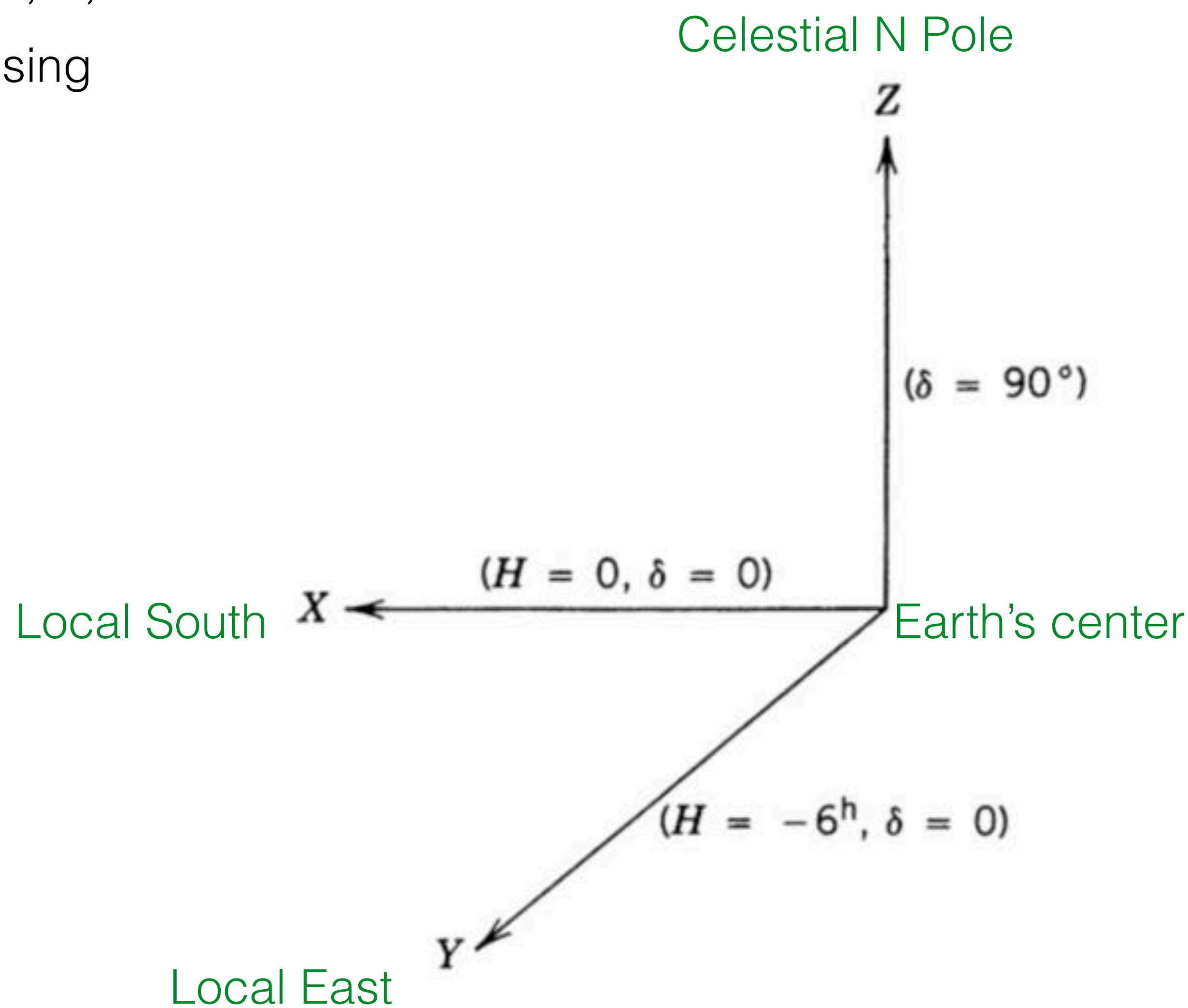
Fourier Sampling

- Of course, interferometers only sample some *uv* spacings
  - Largest *uv* distances determine resolution
  - Inner *uv* hole → Missing large-scale emission (sky is high-pass filtered)



Fourier Sampling

- Baseline coordinates (*u, v, w*) are projection of telescope separation into plane perpendicular to source direction (*u, v*) and toward source (*w*)
- Baseline coordinates: X, Y, Z
  - For VLBI, X Y defined using Greenwich meridian



Fourier Sampling

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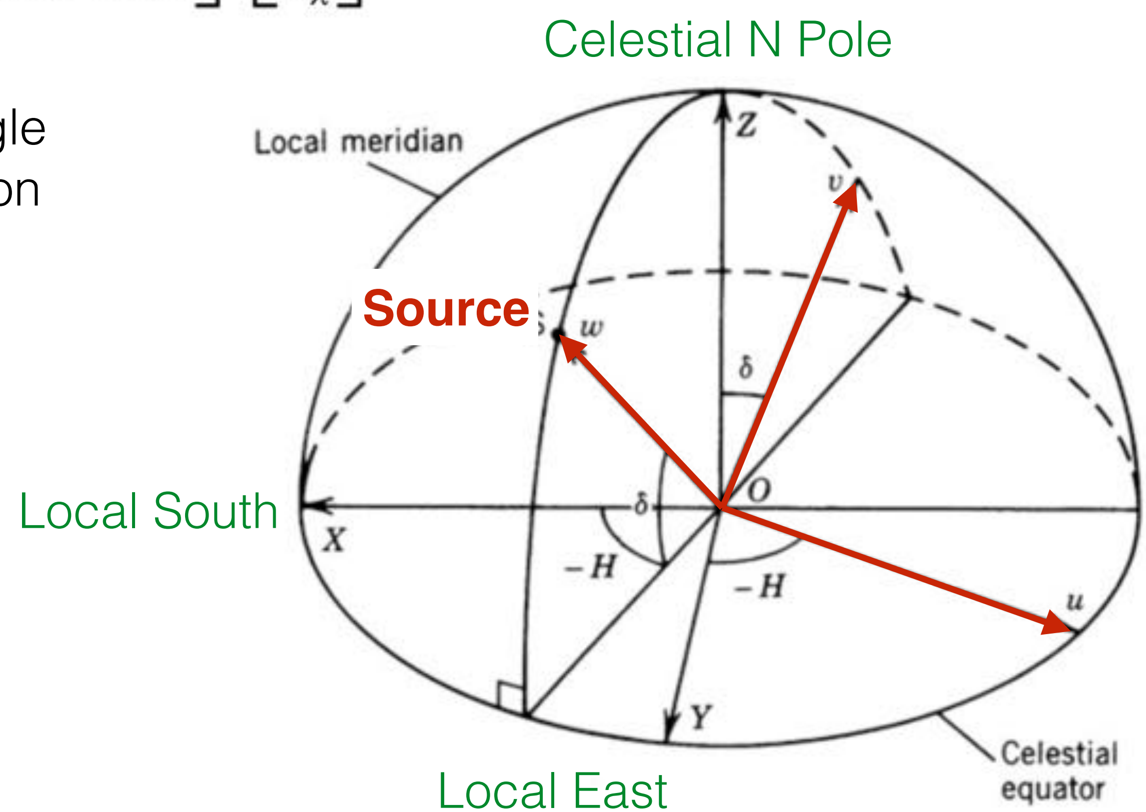
$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} \sin H & \cos H & 0 \\ -\sin\delta\cos H & \sin\delta\sin H & \cos\delta \\ \cos\delta\cos H & -\cos\delta\sin H & \sin\delta \end{bmatrix} \begin{bmatrix} X_\lambda \\ Y_\lambda \\ Z_\lambda \end{bmatrix} .$$

H = Hour Angle
δ = Declination



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- From above, *u, v* can be rearranged:

$$u^2 + \left(\frac{v - Z_\lambda \cos\delta_0}{\sin\delta_0} \right)^2 = X_\lambda^2 + Y_\lambda^2 .$$

  - Offset in v : $Z \cos\delta$
  - Ellipsoid axis ratio: $\sin\delta$

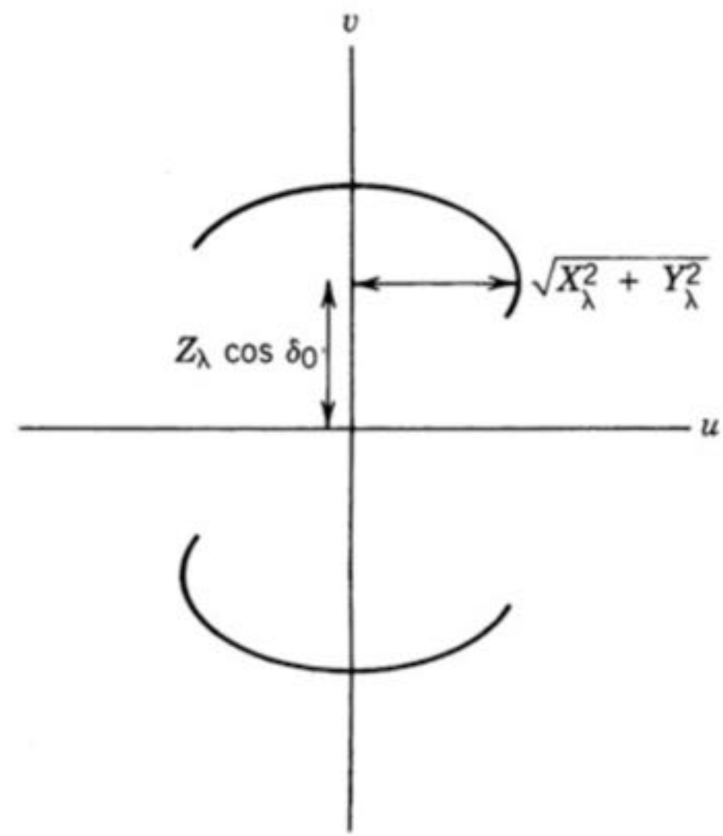
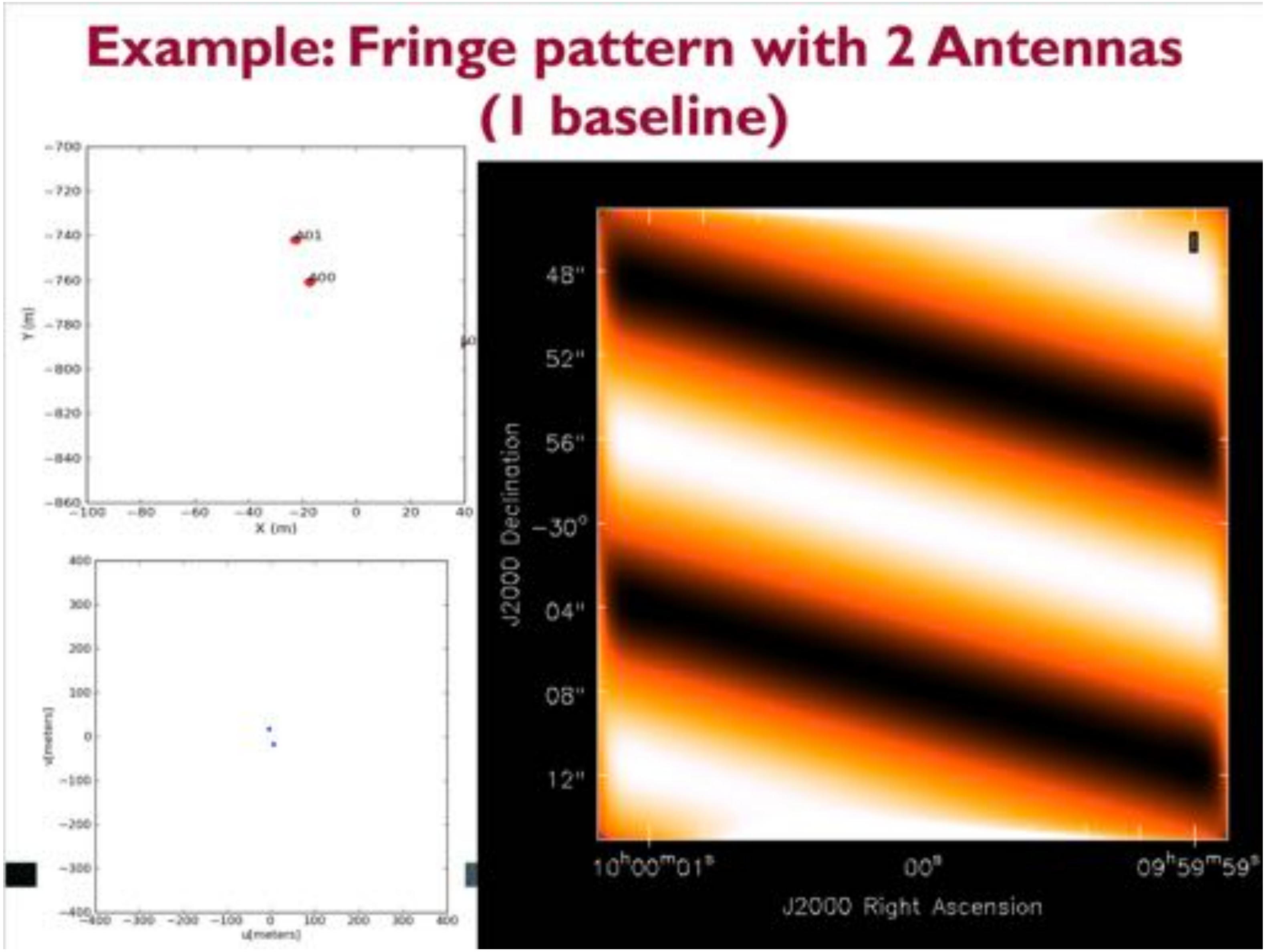


Image Synthesis

- Fourier transform of the uv coverage is your "dirty beam"



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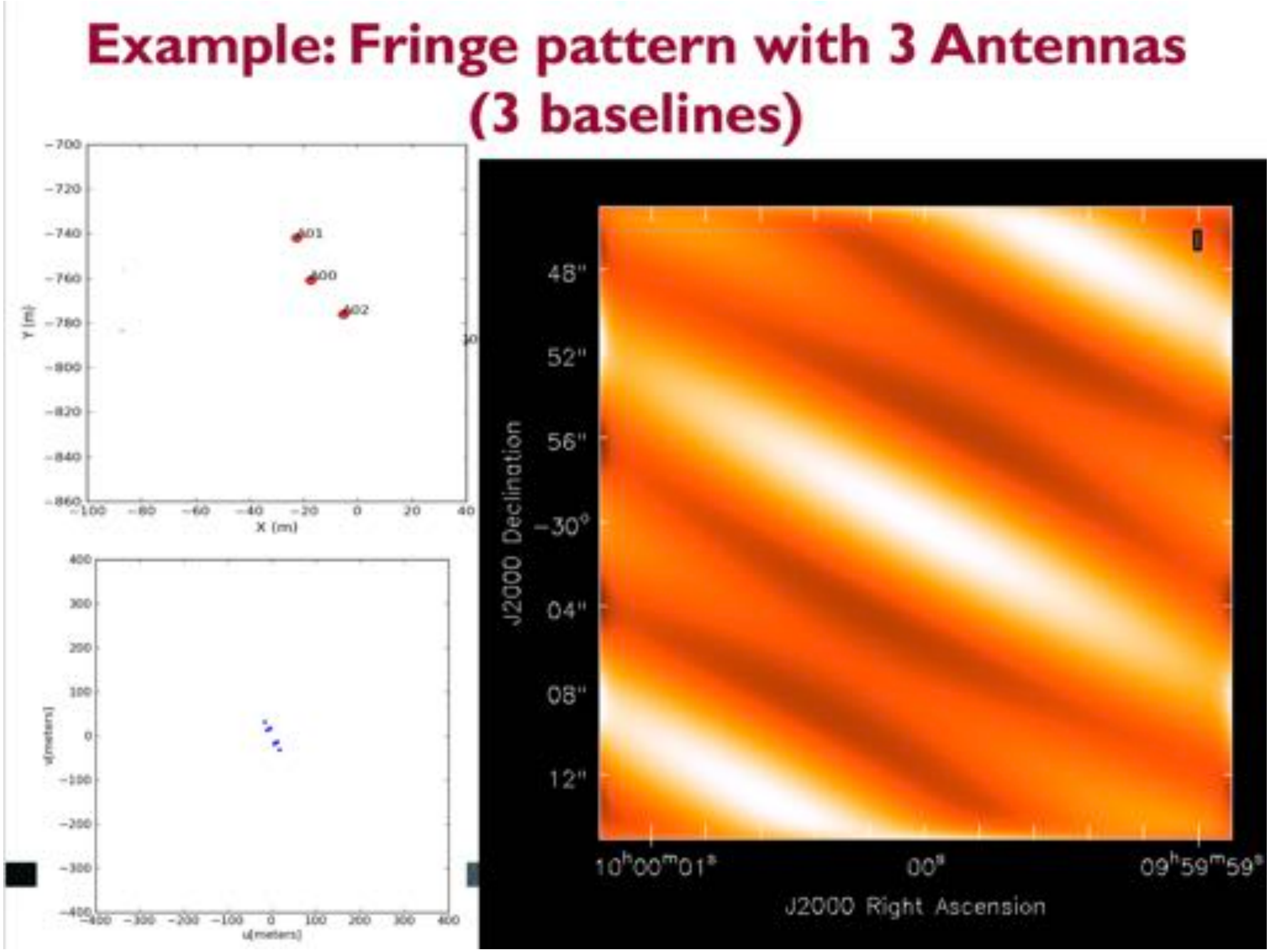

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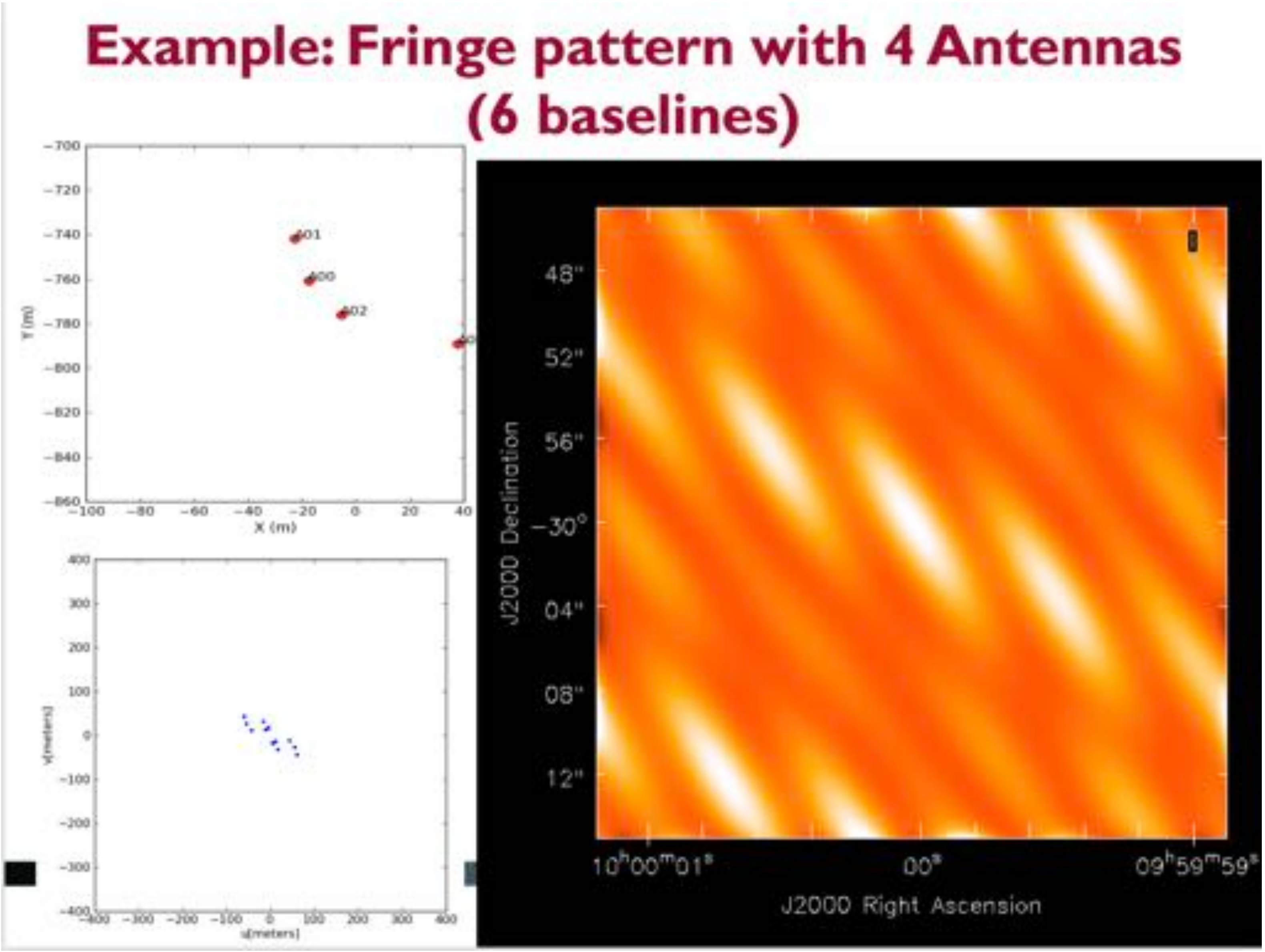

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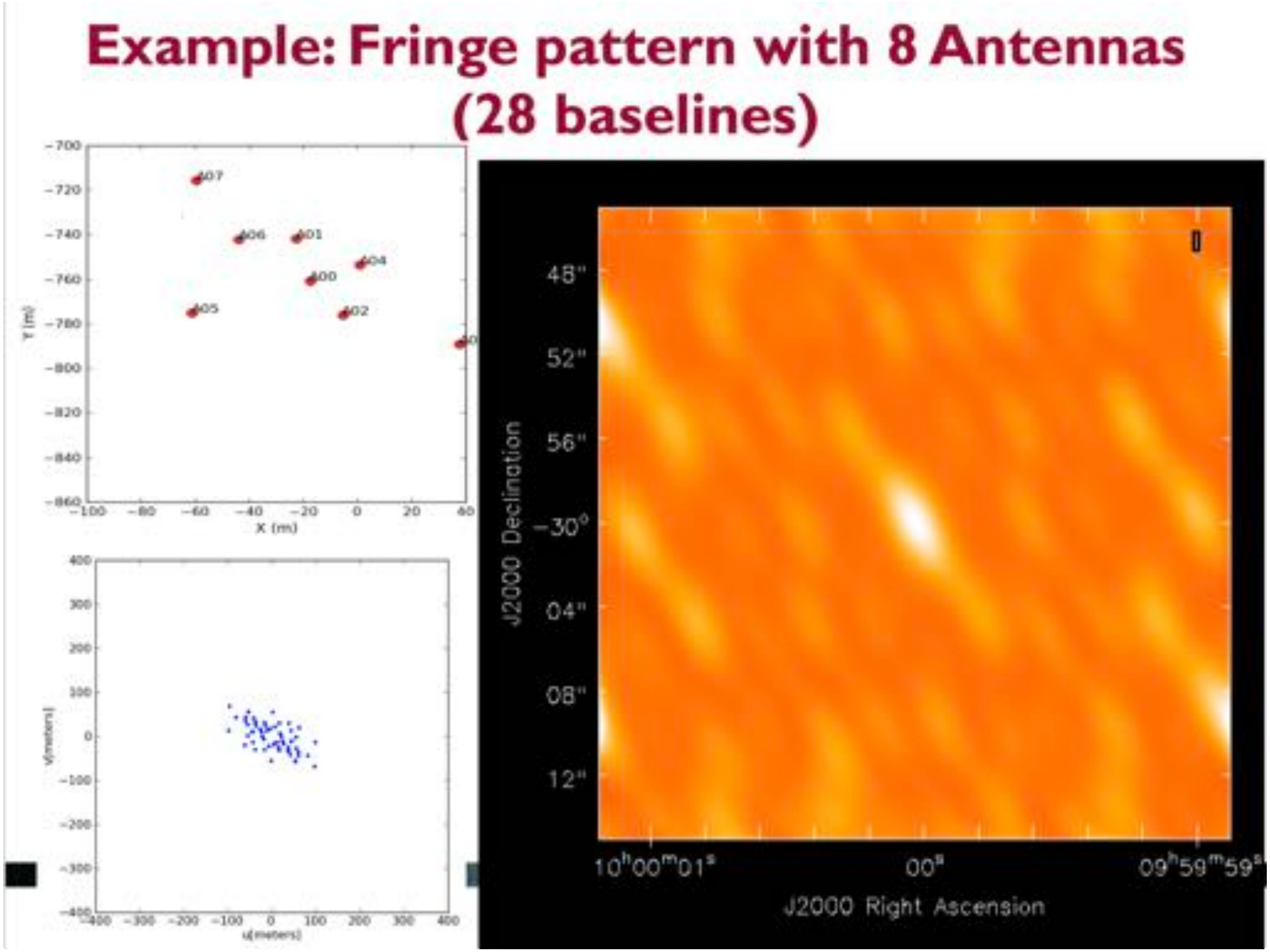

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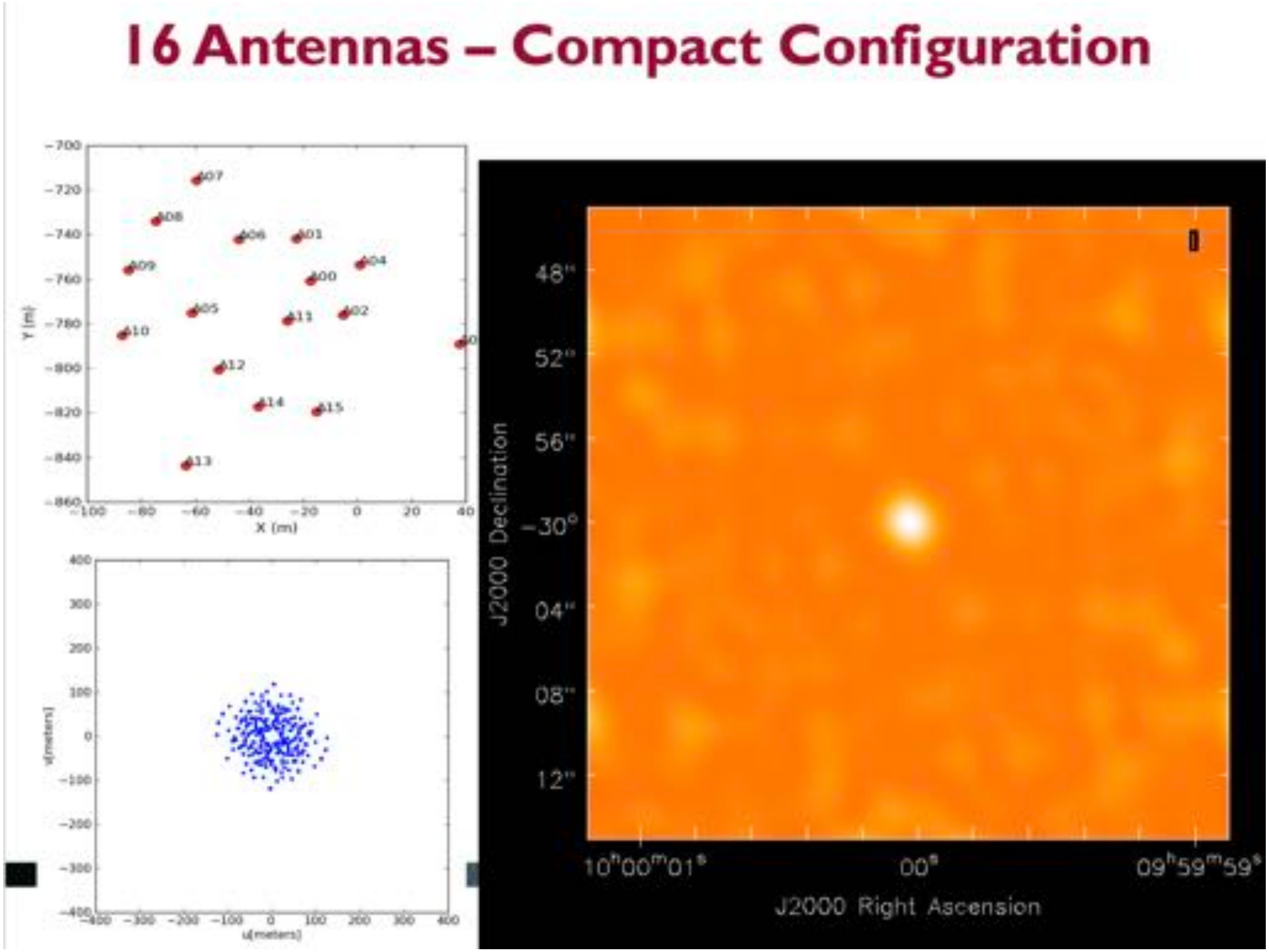

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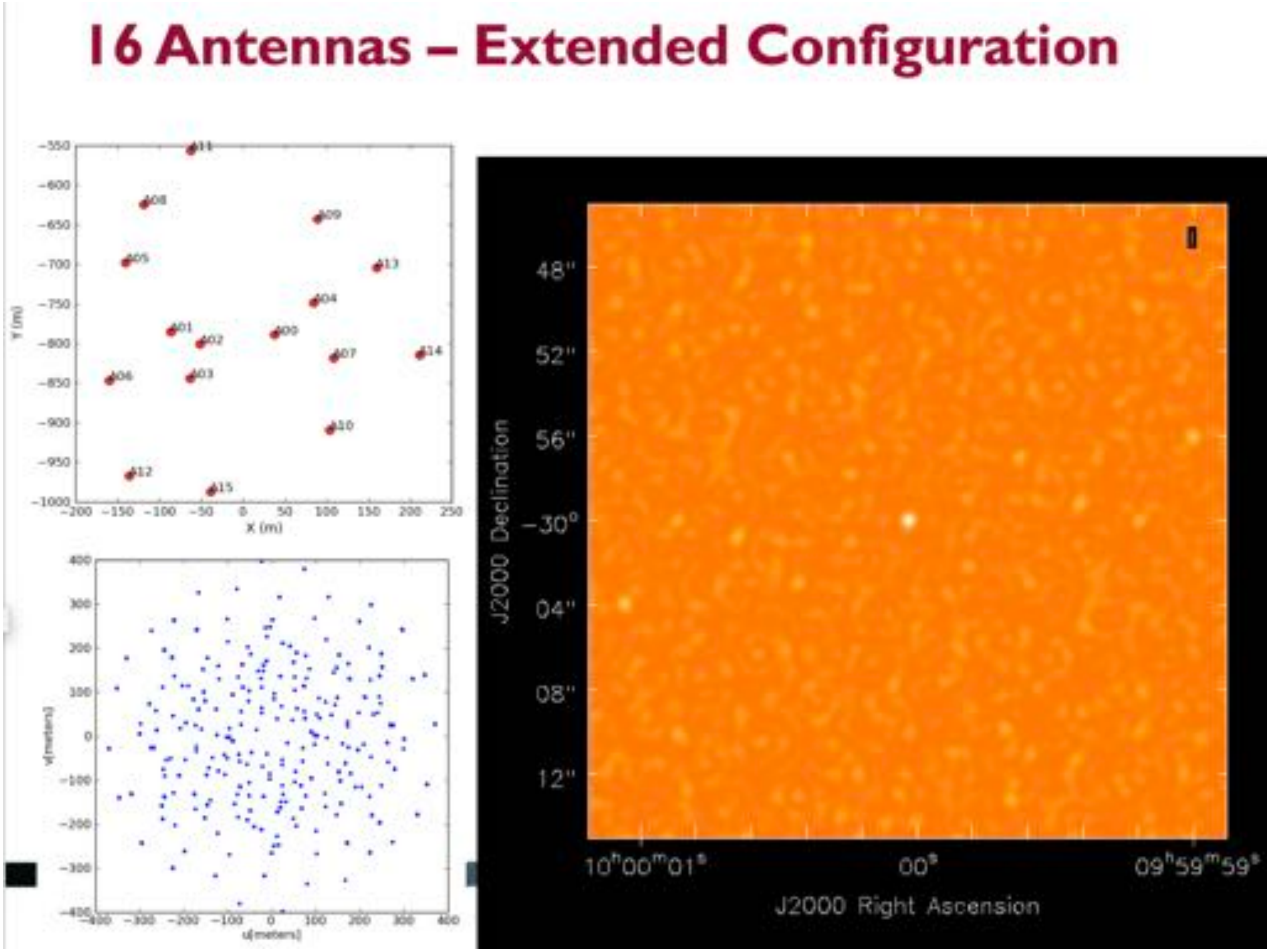


Fourier Relations in Interferometry

- Some sample visibility functions

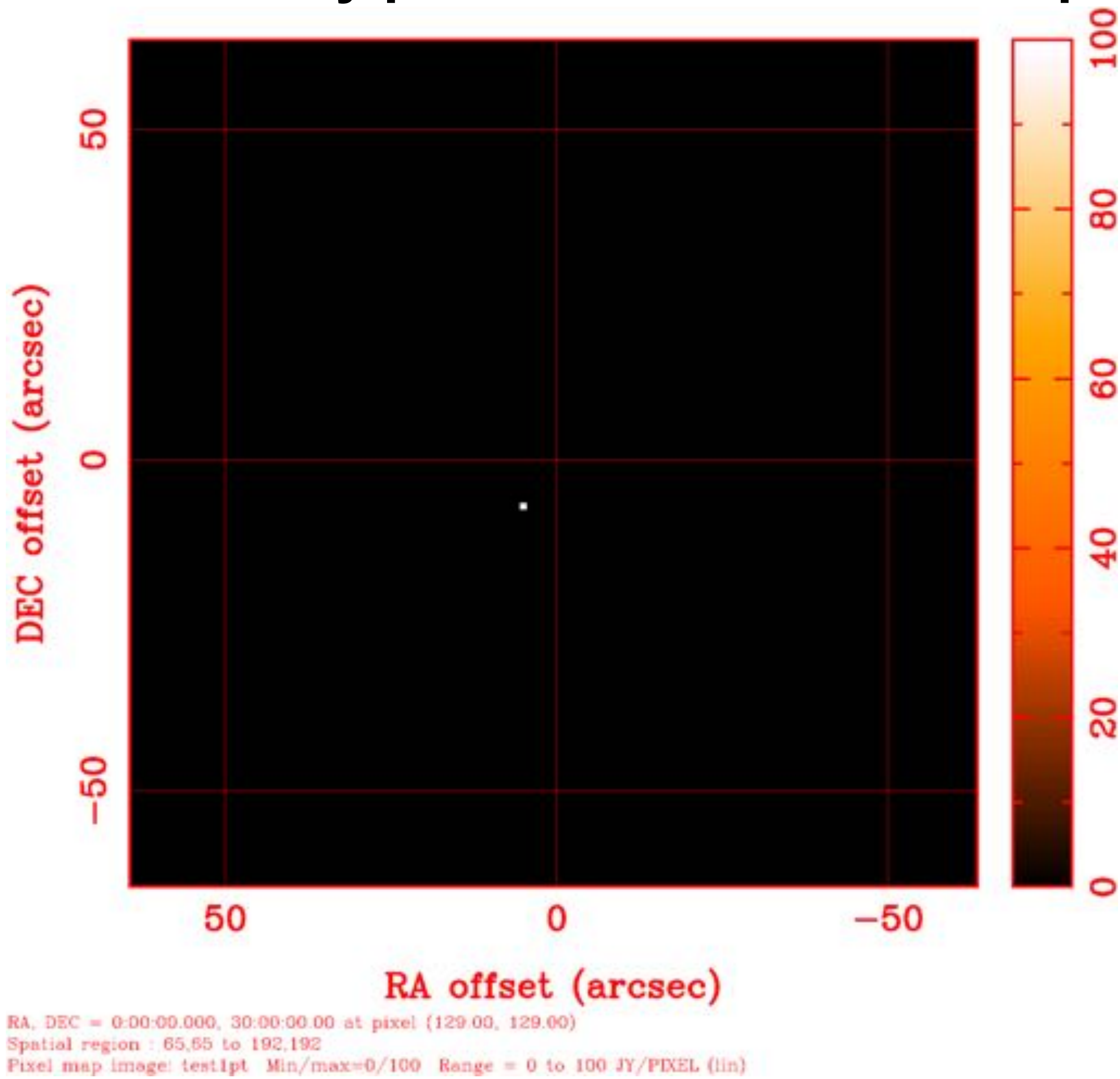
- Reminder: Visibility function is complex number

  - Complex correlator provides in-phase and quadrature-phase outputs, giving the real (cosine) and imaginary (sine) components

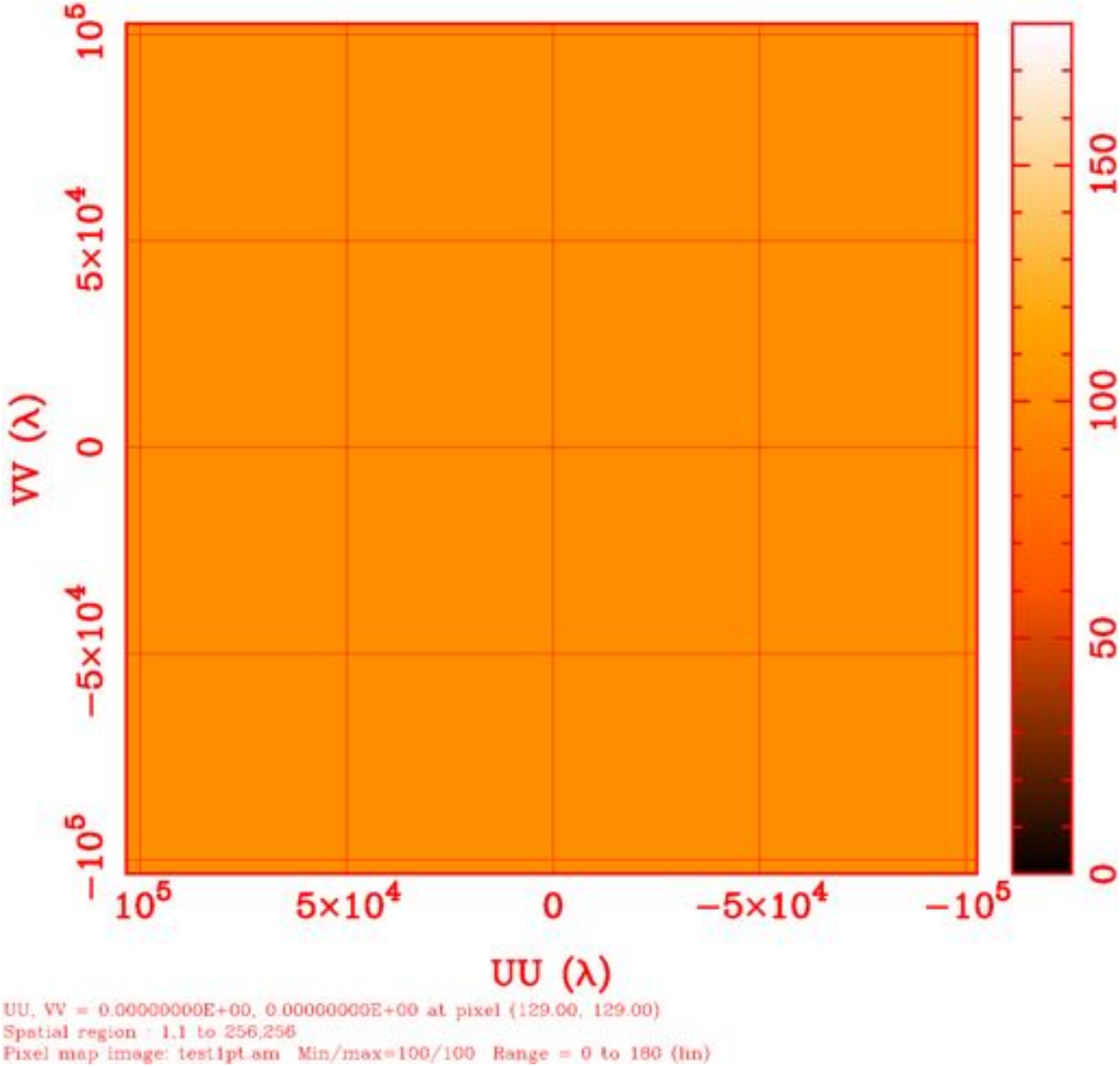
  - Can also be expressed as amplitude and phase

$$V(u, v) = V_R + i\,V_I = |V|\,e^{i\theta}$$

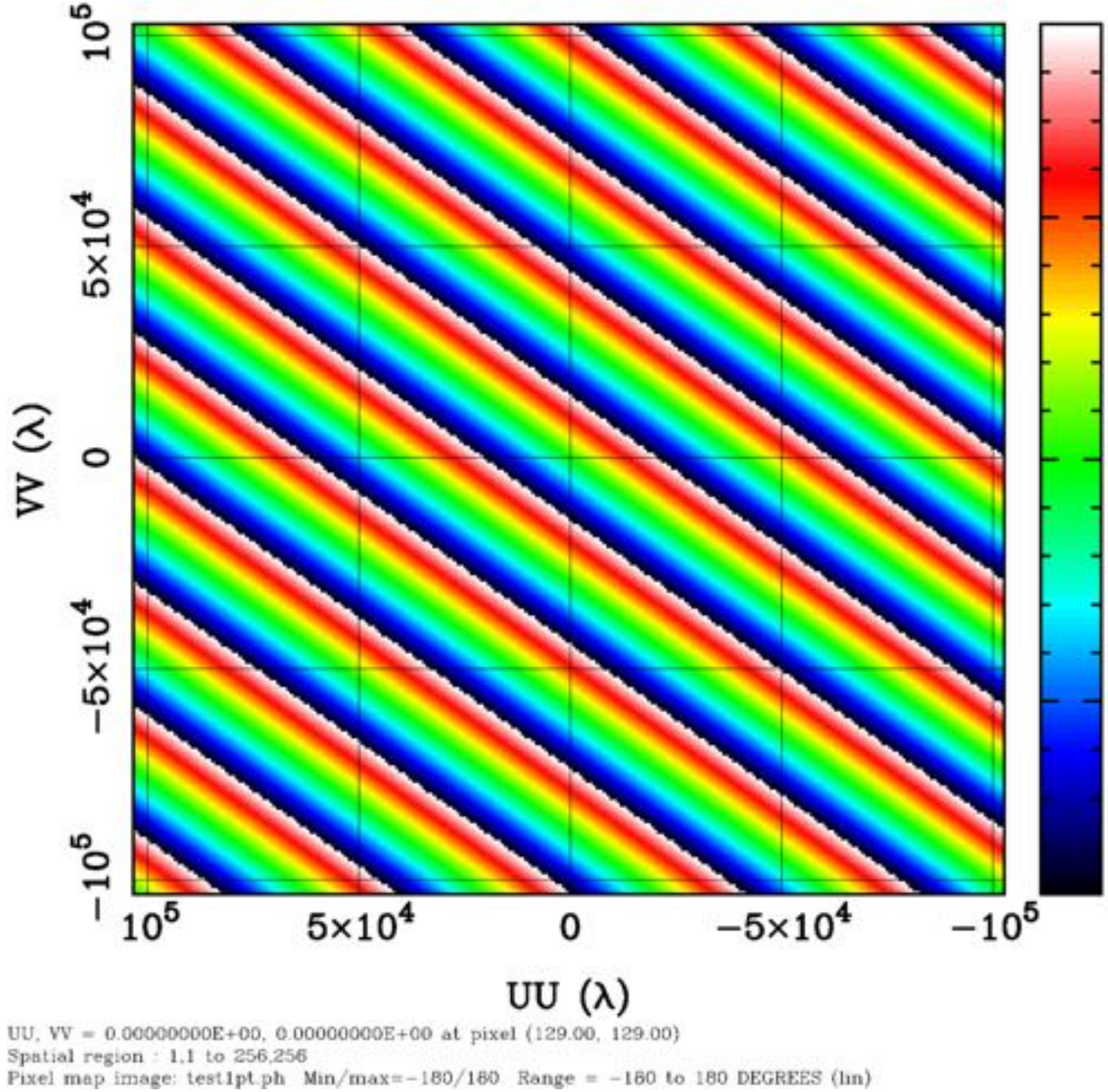
100 Jansky point source offset from phase center



Visibility amplitude for 100 Jy point source

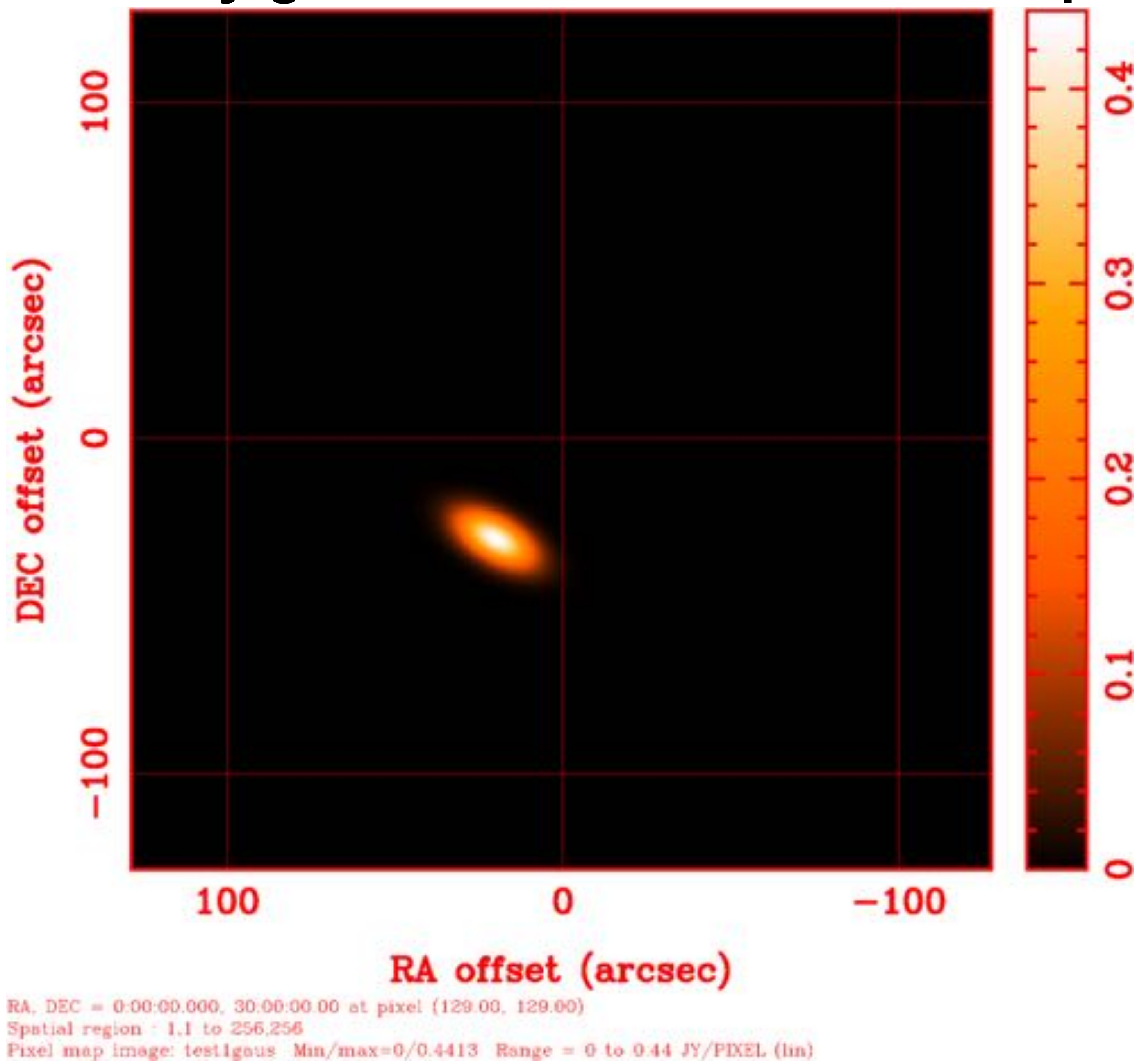


Visibility phase for offset 100 Jy point source

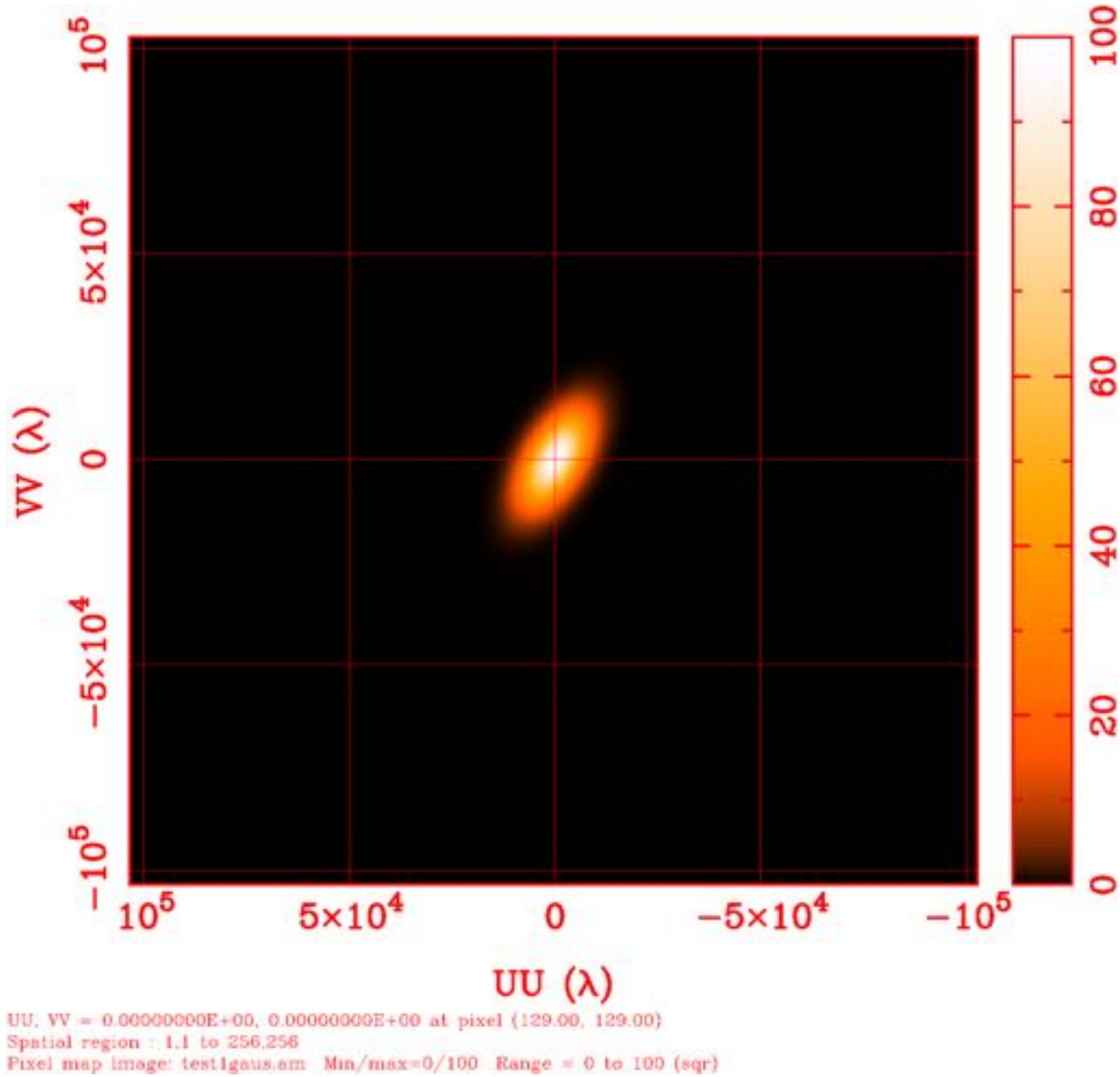


spacing between cycles is inversely prop. to distance of source from phase center

100 Jy gaussian source offset from phase center

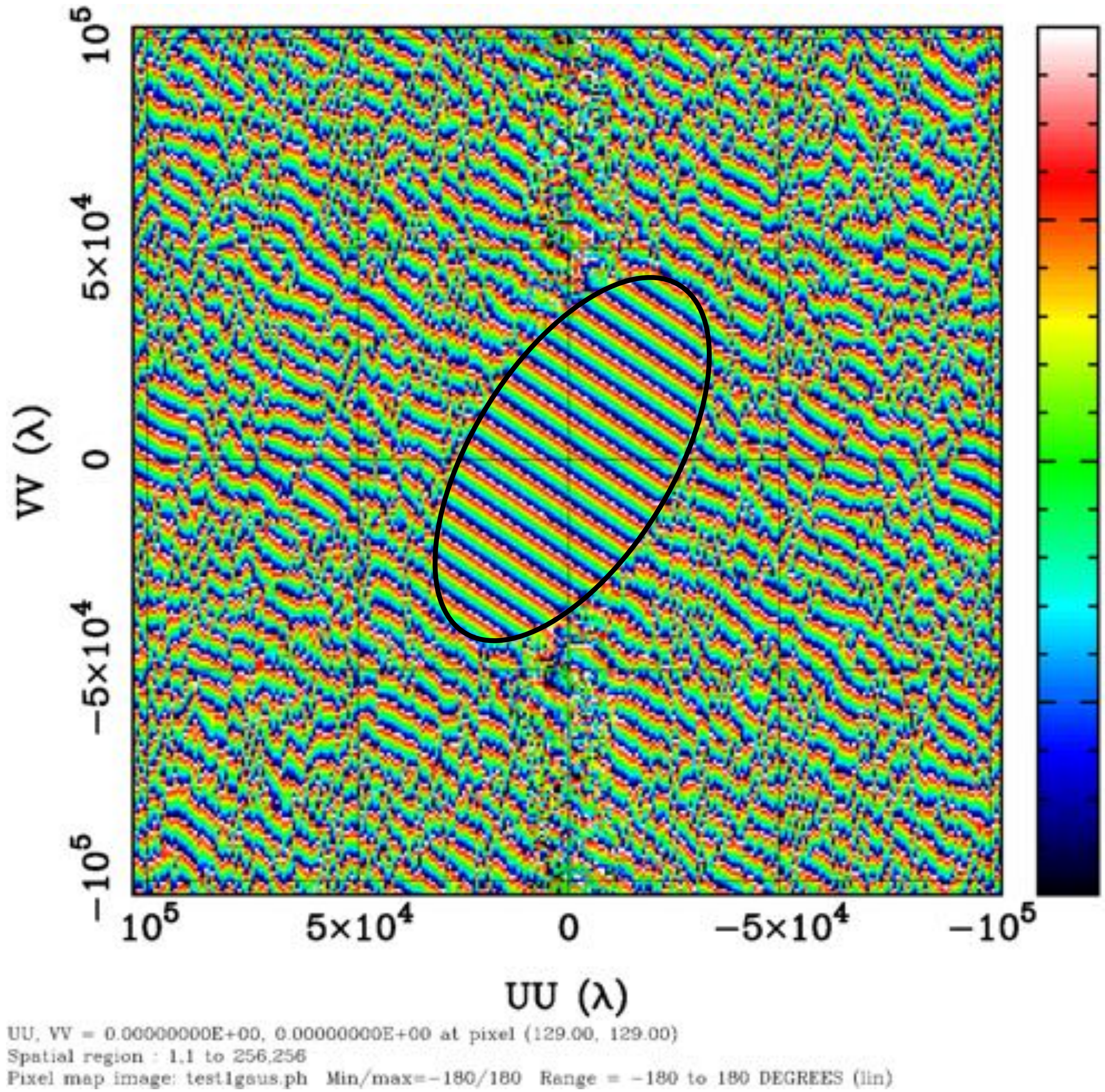


Visibility amplitude for 100 Jy gaussian source



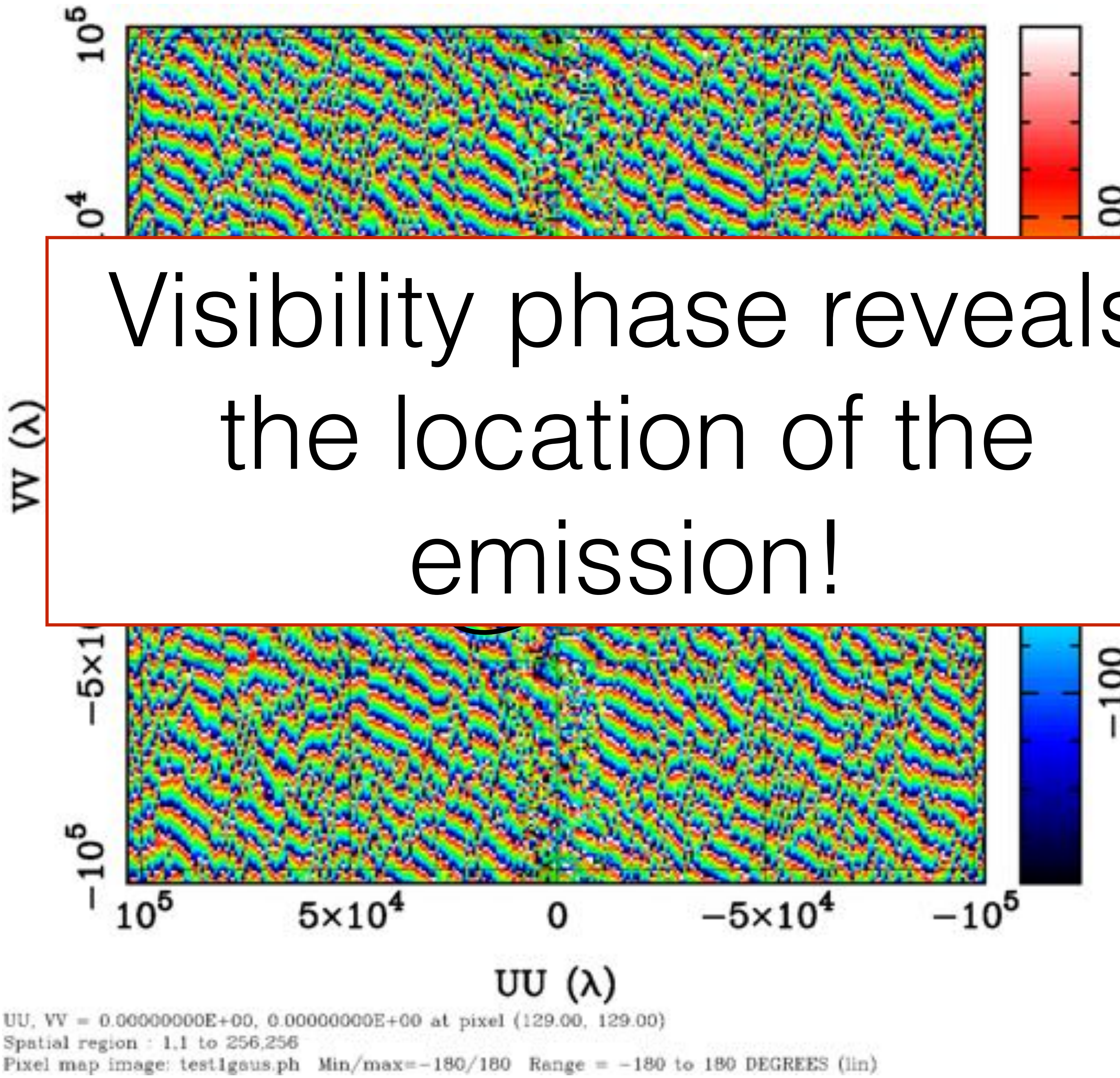
Visibility amplitude at (U,V)=(0,0) is total flux in source

Visibility phase for offset 100 Jy gaussian source



Deviations from straight lines are due to computational rounding where visibility amplitude is very low

Visibility phase for offset 100 Jy gaussian source



Visibility phase reveals the location of the emission!

Deviations from straight
...es are due
...
...omputational
...unding
...here visibility amplitude is very low

Roomba

Shep

$|\text{Shep}| * \exp(i*\phi_{\text{Roomba}})$

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Roomba

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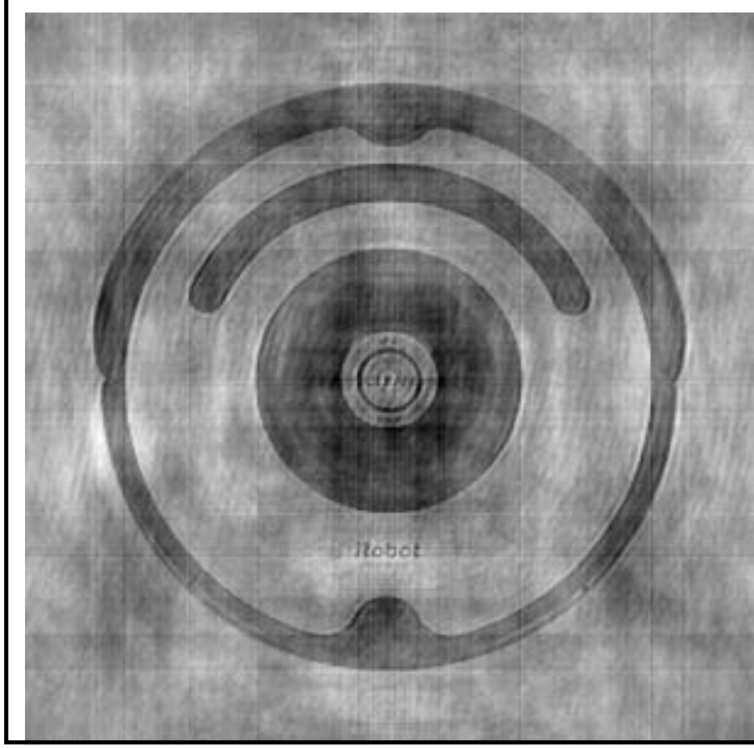
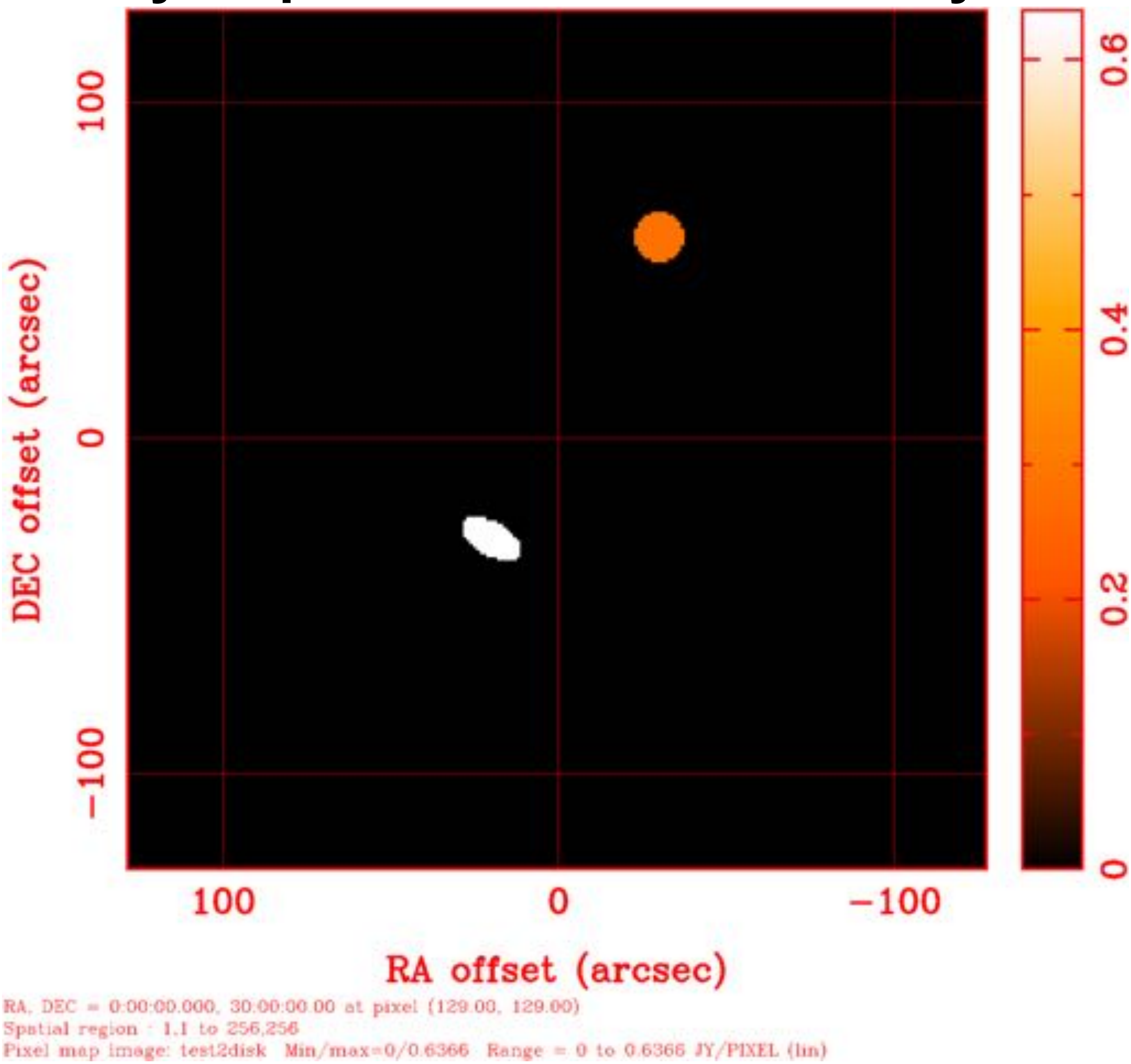
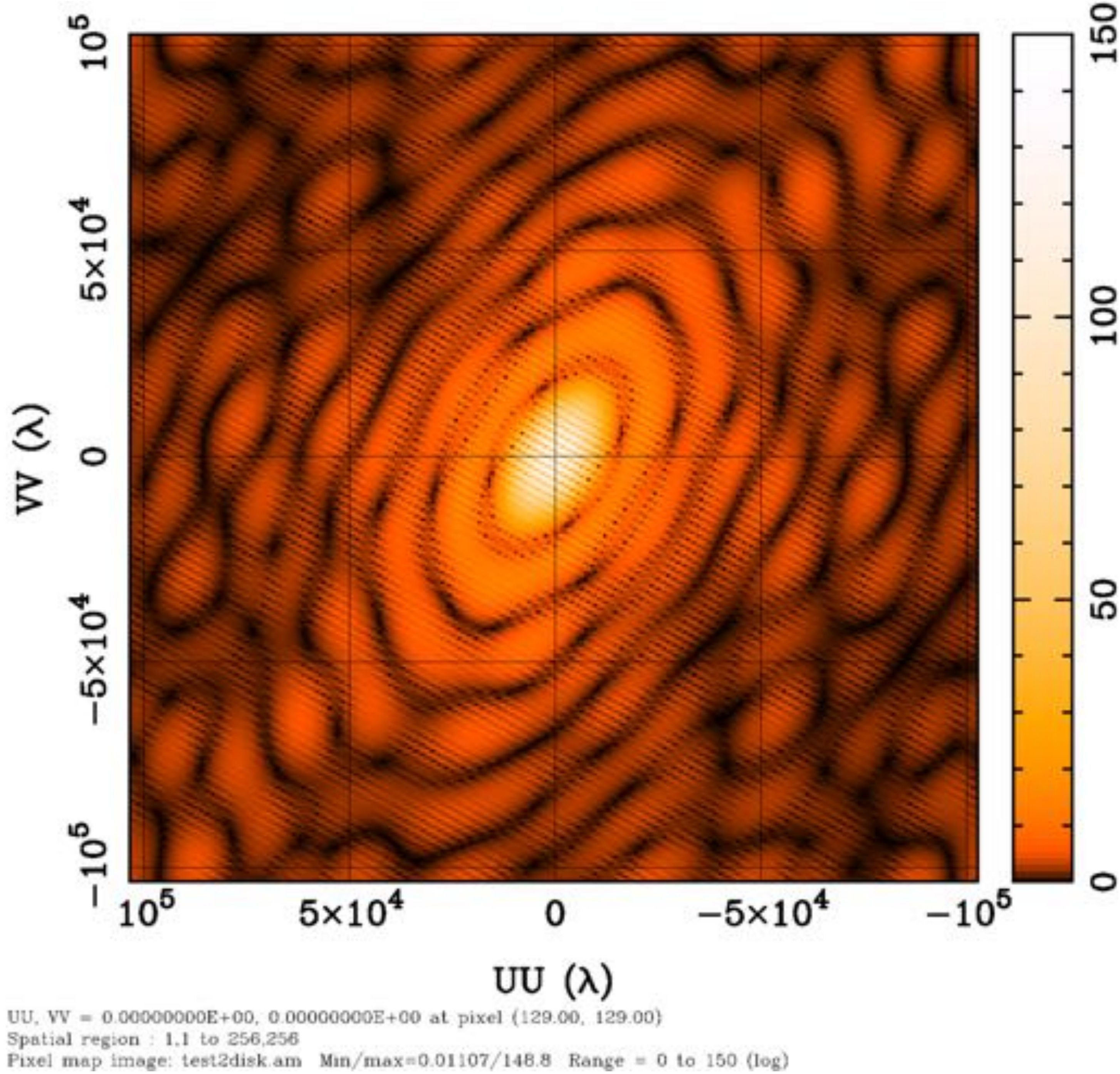


$|\text{Roomba}| * \exp(i*\phi_{\text{Shep}})$

100 Jy elliptical unif. disk and 50 Jy circular unif. disk



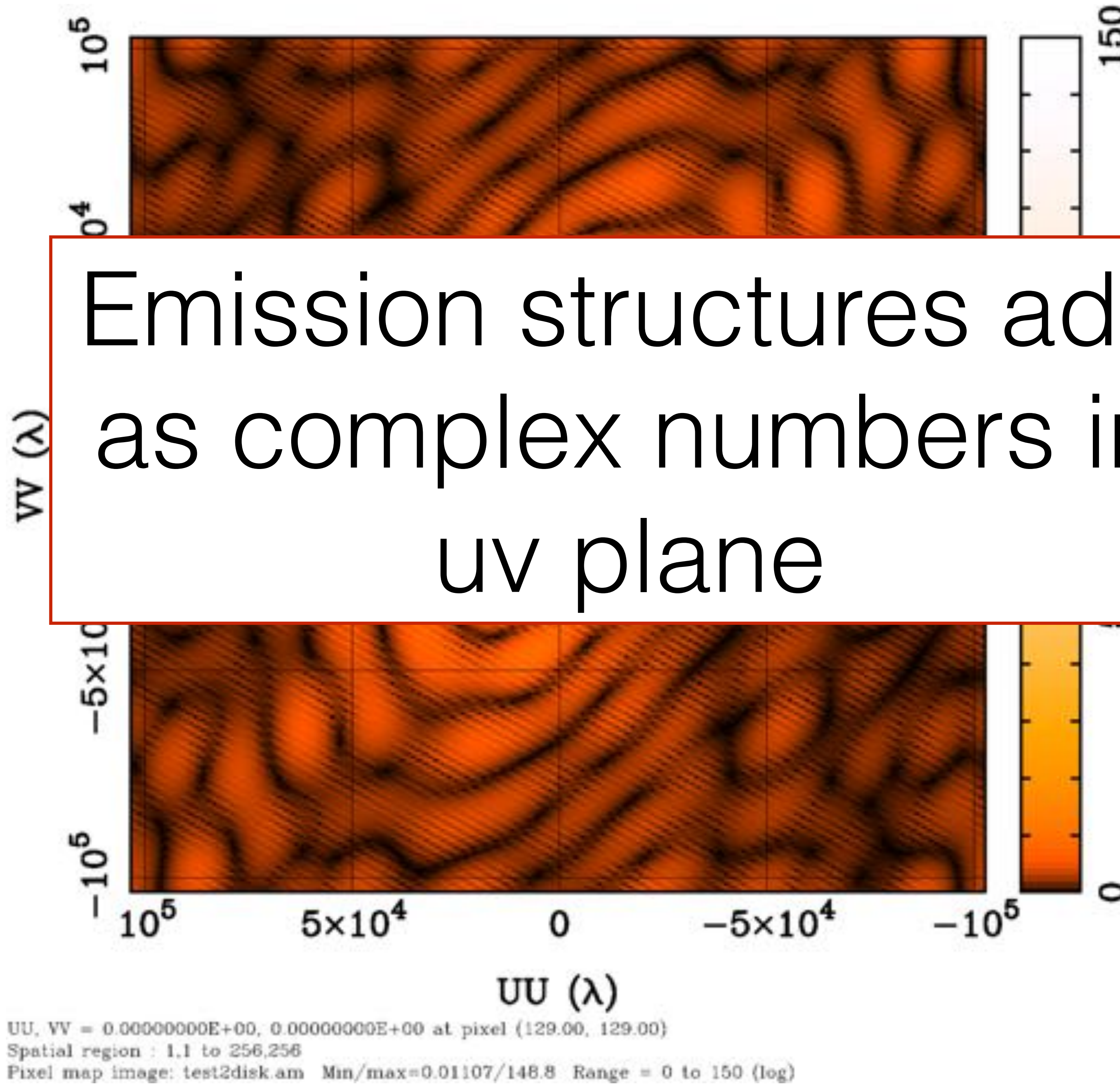
Visibility amplitude for elliptical plus circular disks



Visibility amplitude at (U,V)=(0,0) is total flux from both sources, 150 Jy.

Sharp edges in source create lots of high spatial frequency content (note log stretch in color wedge)

Visibility amplitude for elliptical plus circular disks



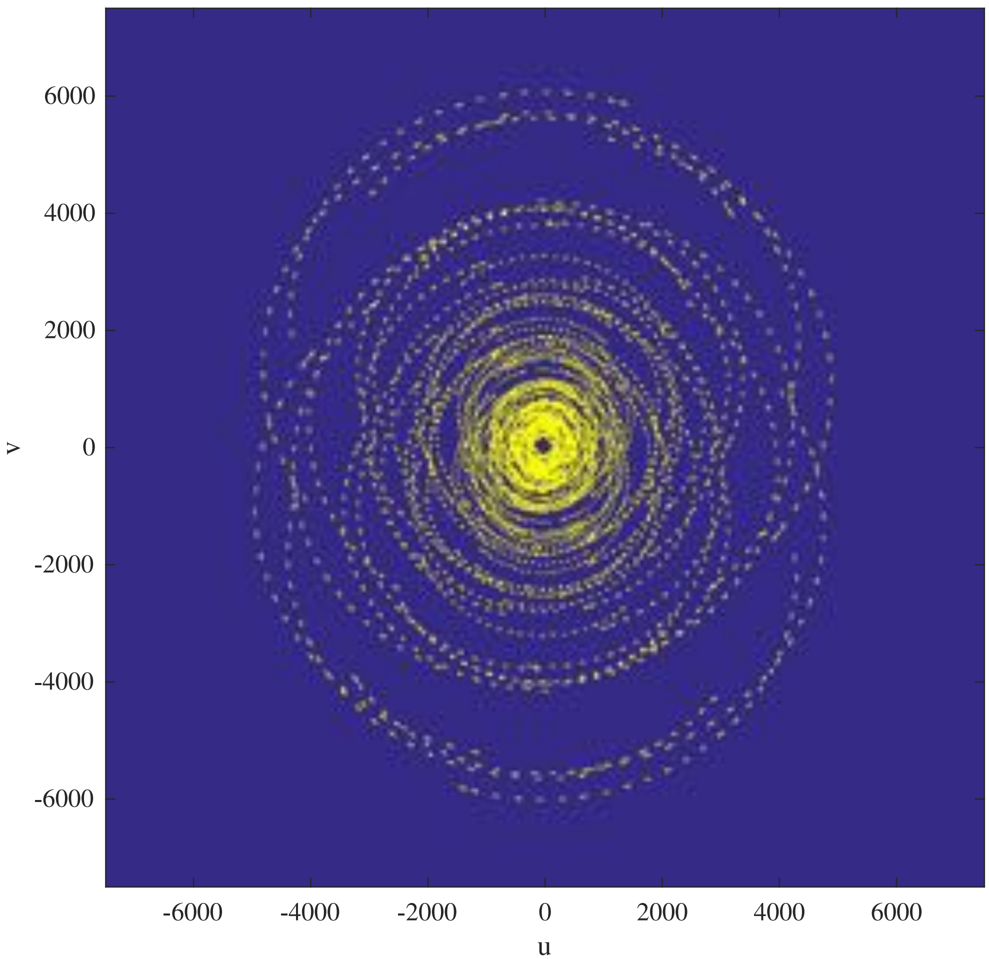
Emission structures add as complex numbers in uv plane

Visibility amplitude at (,V)=(0,0) is tal flux from th sources, 0 Jy.

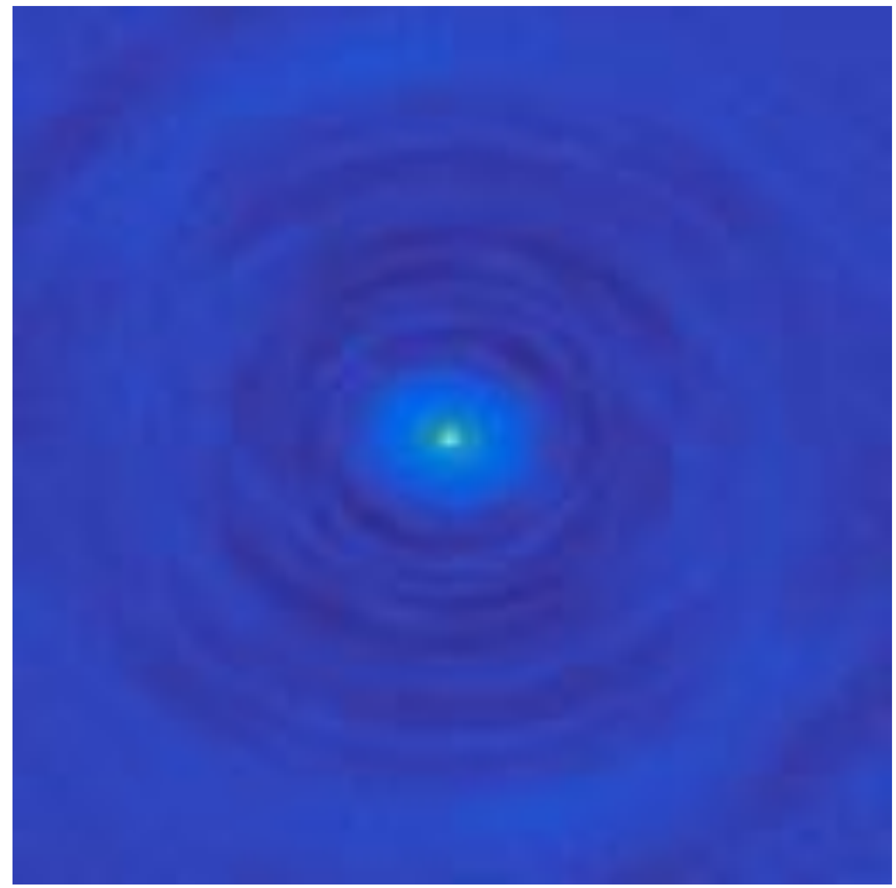
harp edges in urce create lots of high spatial frequency content (note log stretch in color wedge)

Fourier Relations in Interferometry

- Fourier transform of the uv coverage is your "dirty beam"



$\rightleftharpoons$

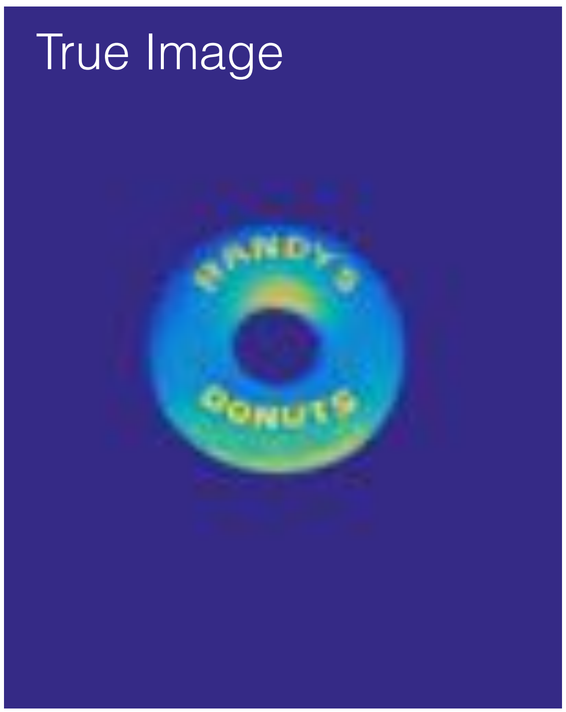


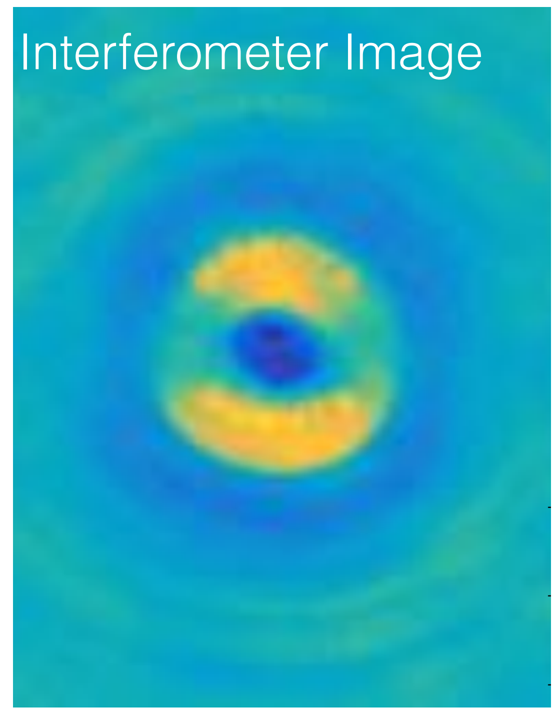
Fourier Relations in Interferometry

- uv coverage (FT of beam) **multiplies** 2D visibility function (FT of image)
  So "dirty beam" is **convolved** with true image

Fourier Relations in Interferometry

- uv coverage (FT of beam) **multiplies** 2D visibility function (FT of image)
  So beam is **convolved** with true image
- Deconvolution techniques are central to understanding these "images"





Closing Remarks

- Fill out the evaluation!
  bit.ly/BH_Interferometry_Survey

- Future webinars!
  - March 11: VLBI Data Series: Session 1 - Handling Data, Managing Errors
  - March 18: VLBI Data Series: Session 2 - Imaging Techniques
  - May 5: VLBI Data Series Session 3 - Model Comparison