Ray Tracing:

Interfacing Black Hole Theory and Observations

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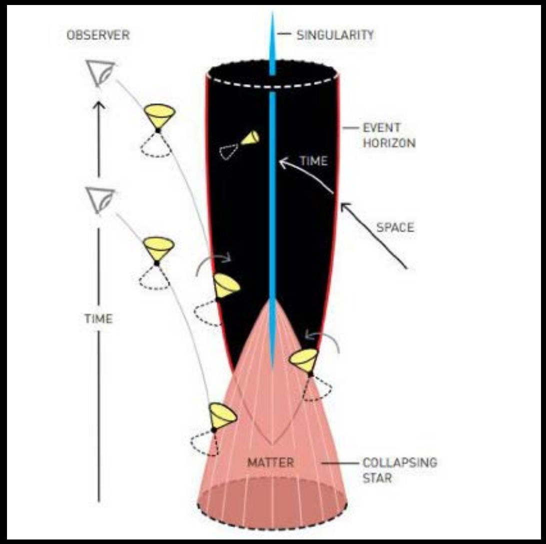
Oct 27th, 2020, PIRE Webinar

Overview

Black Hole Accretion Physics (Narayan)

General Relativity (Berti) + Fluid Mechanics (Gammie) + Plasma Physics (Quataert) + Ray Tracing (Chan)



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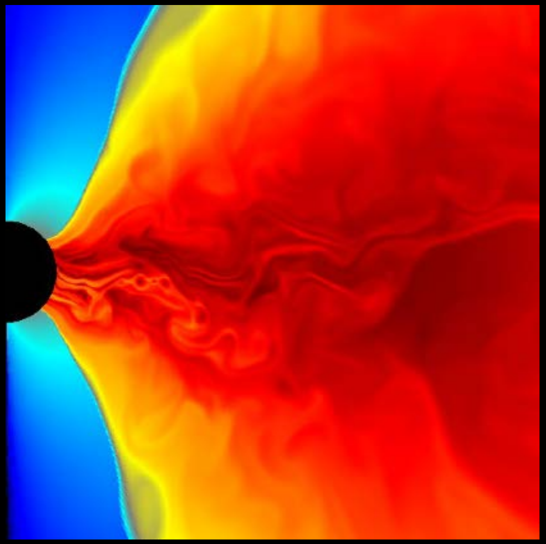

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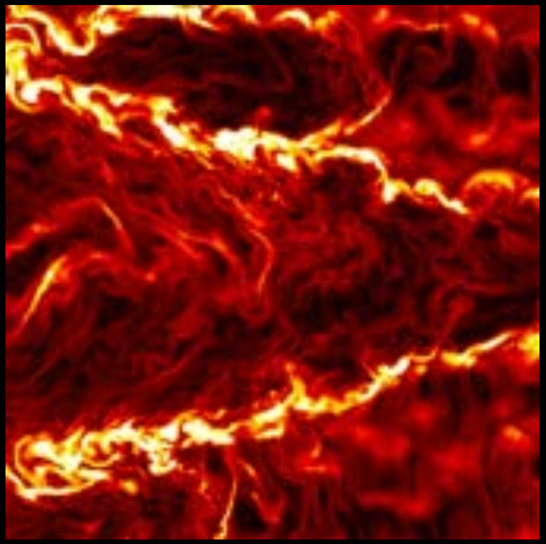

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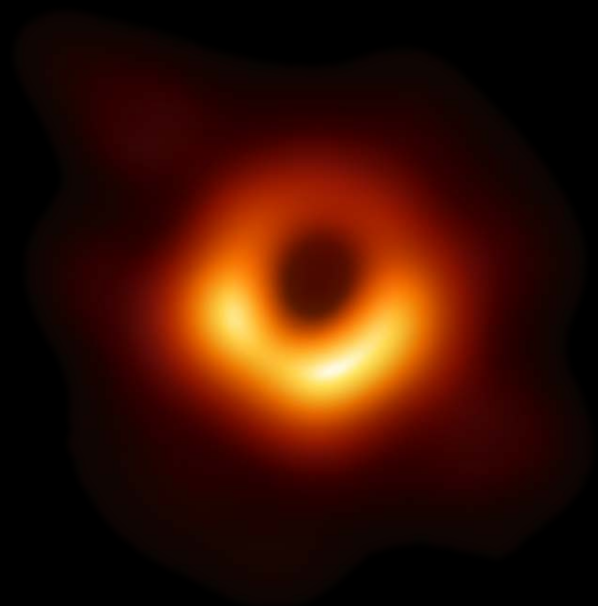
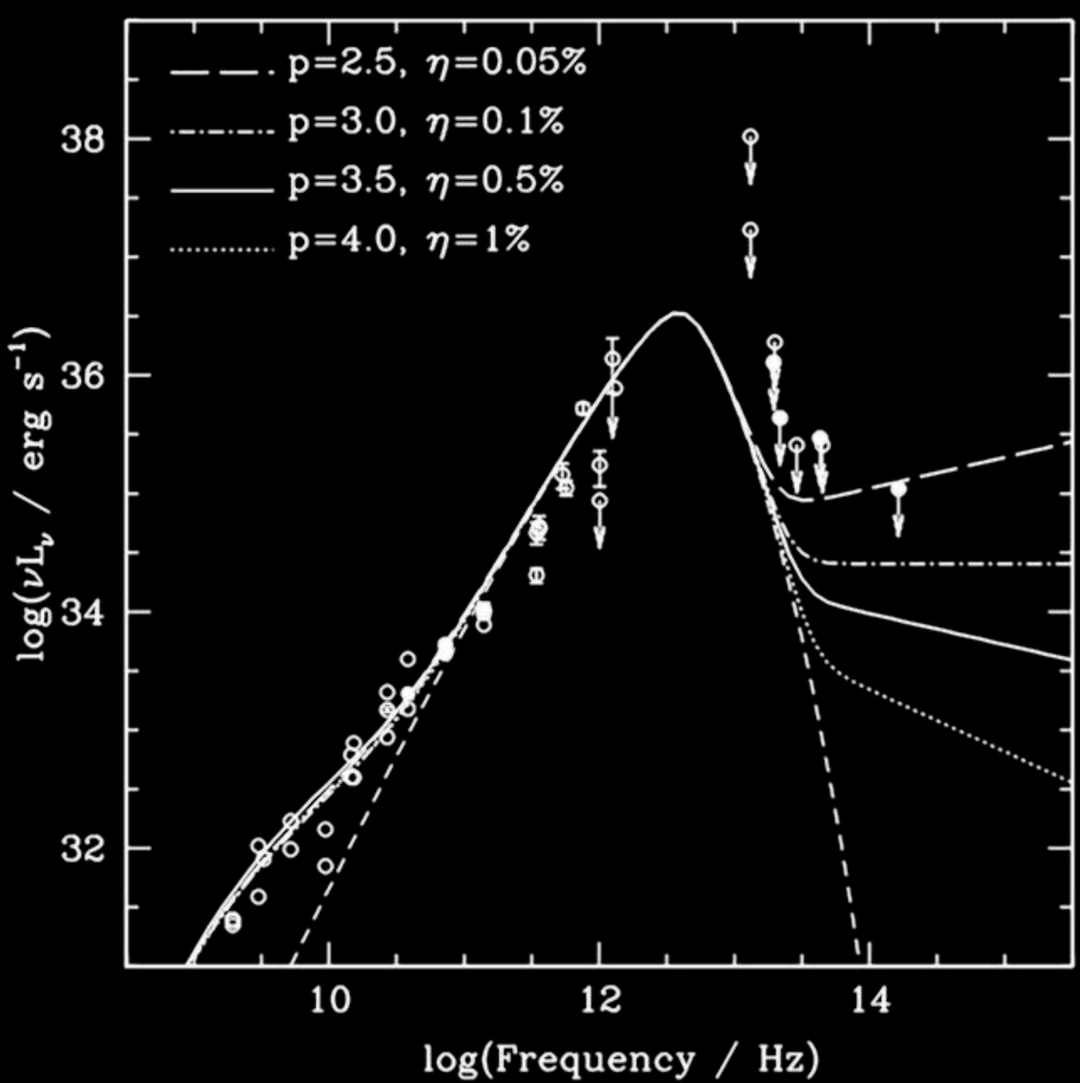


Observations (VLBI Data Series)

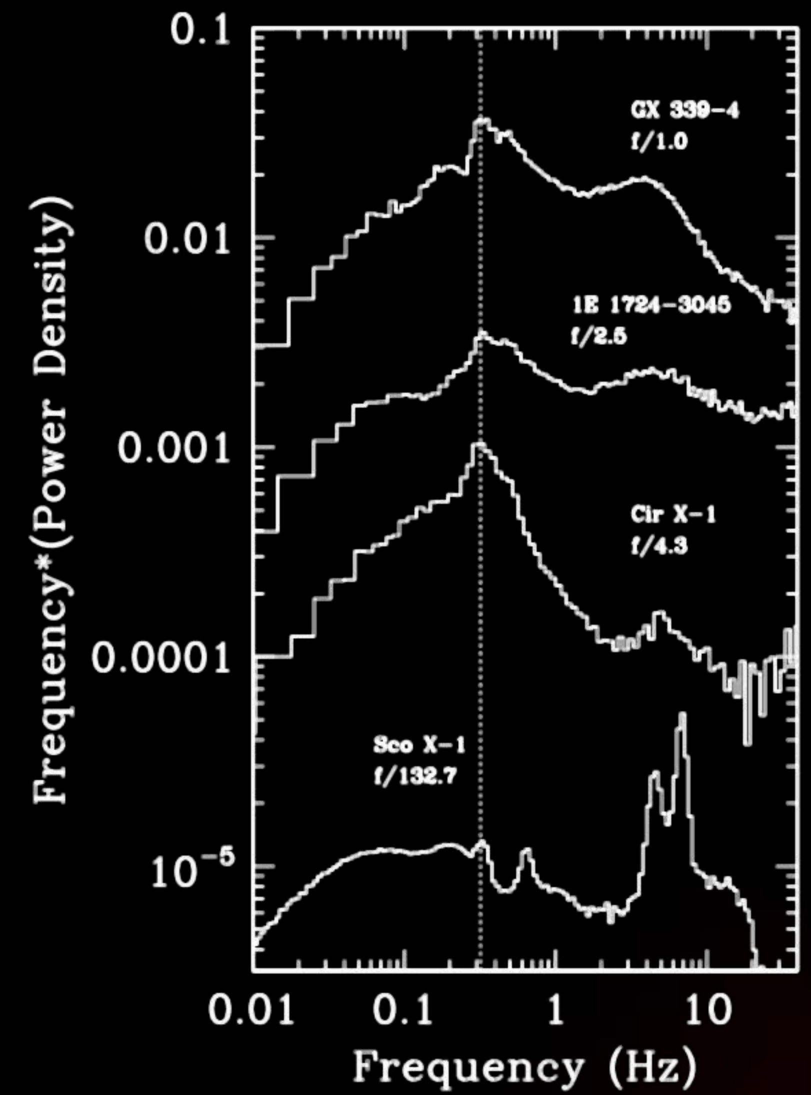
Black Hole PIRE

Non-Image Black Hole Data



Özel, Psaltis, & Narayan (2000)

FIG. 4.—Comparison of spectral models for Sgr A* with radio and IR data. The dashed curve shows the spectrum when the electrons are purely thermal. The other four curves show spectra from hybrid populations with the following combinations of parameters: $p = 2.5$, $\eta = 0.05\%$; $p = 3.0$, $\eta = 0.2\%$; $p = 3.5$, $\eta = 0.5\%$; and $p = 4.0$, $\eta = 1\%$.



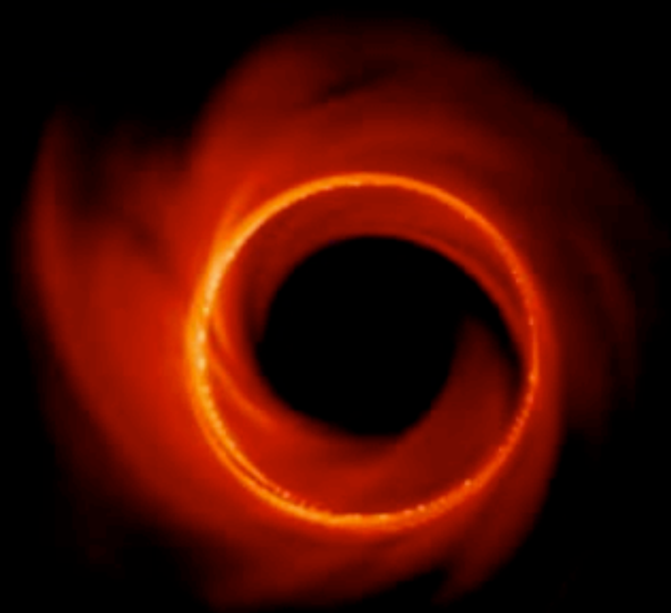
Psaltis, Belloni, & van der Klis (1999)

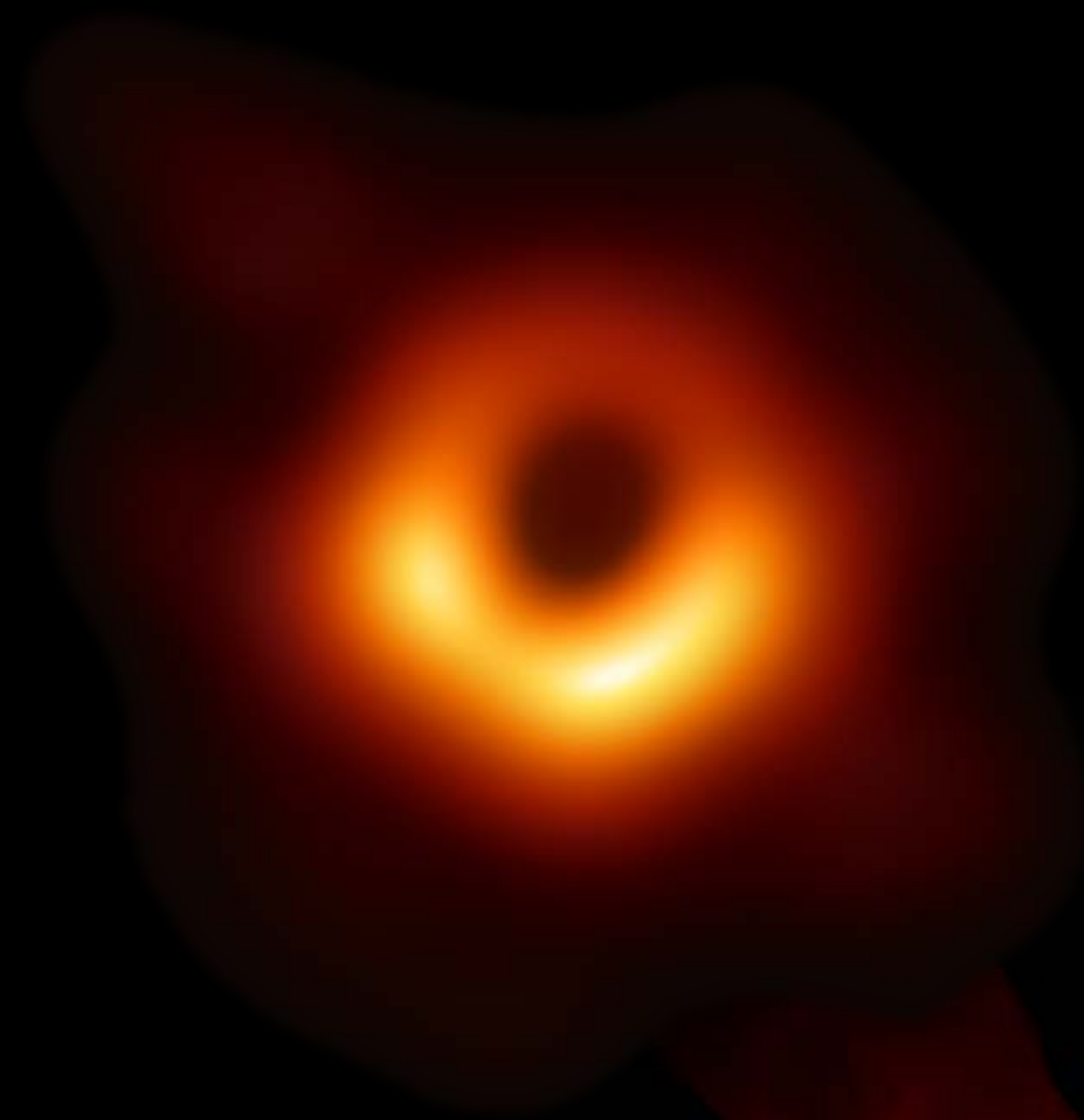
FIG. 1.—Examples of power spectra of low-mass X-ray binaries, in which more than one QPO or broadband noise components are detected. The individual power spectra were shifted along the vertical axis for clarity and along the horizontal axis, by the amounts displayed, for the low-frequency QPOs to be aligned (*dotted line*). The sample of sources includes a black hole candidate (GX 339−4; Méndez et al. 1998), an X-ray burster (1E 1724−3045; Olive et al. 1998), a luminous neutron star (Cir X-1; Shirey 1998), and a Z source (Sco X-1). The continuum in the power spectrum of Sco X-1 at high frequencies is affected by instrumental effects.

The Event Horizon Telescope

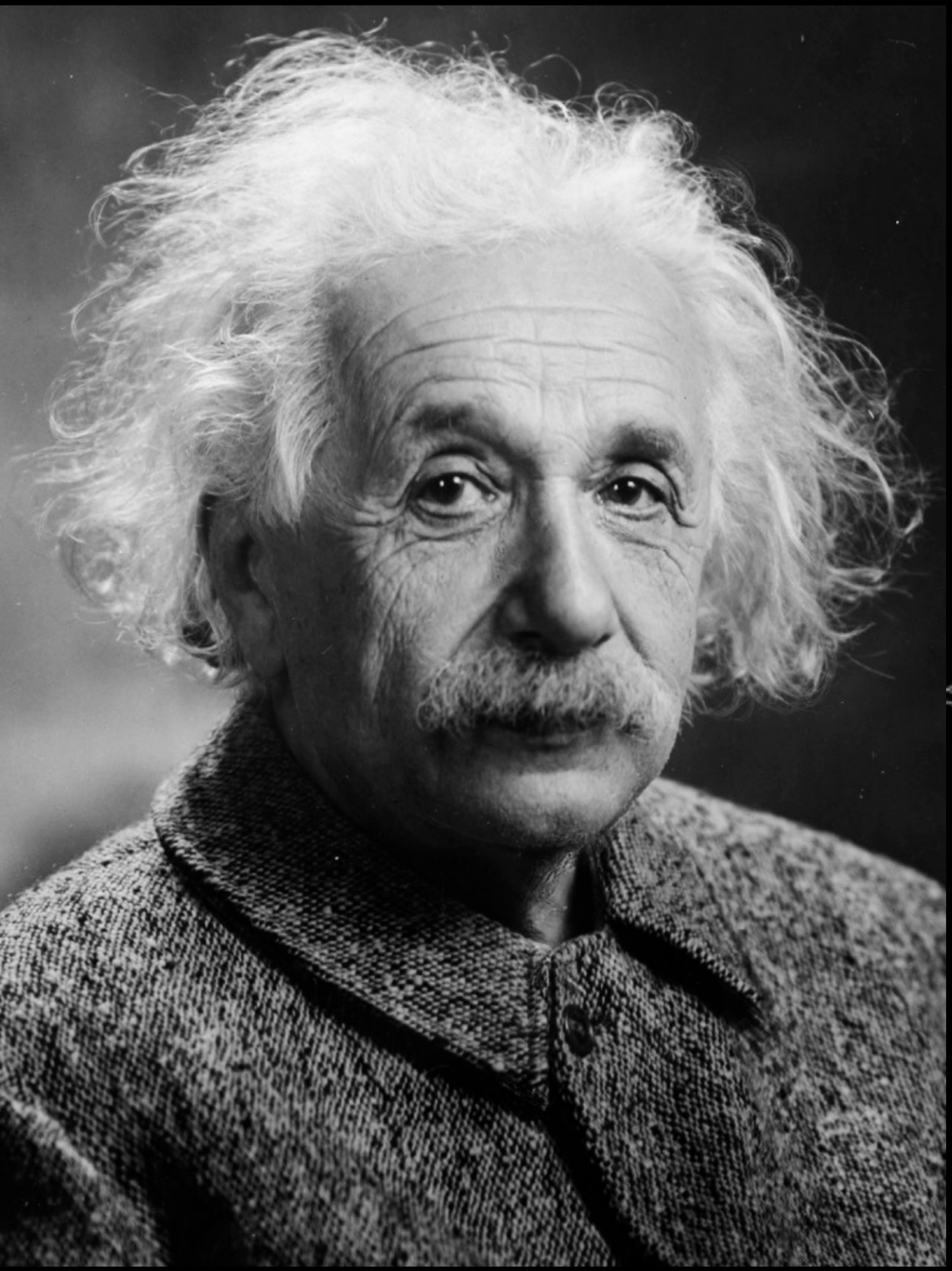
Credit: D Marrone

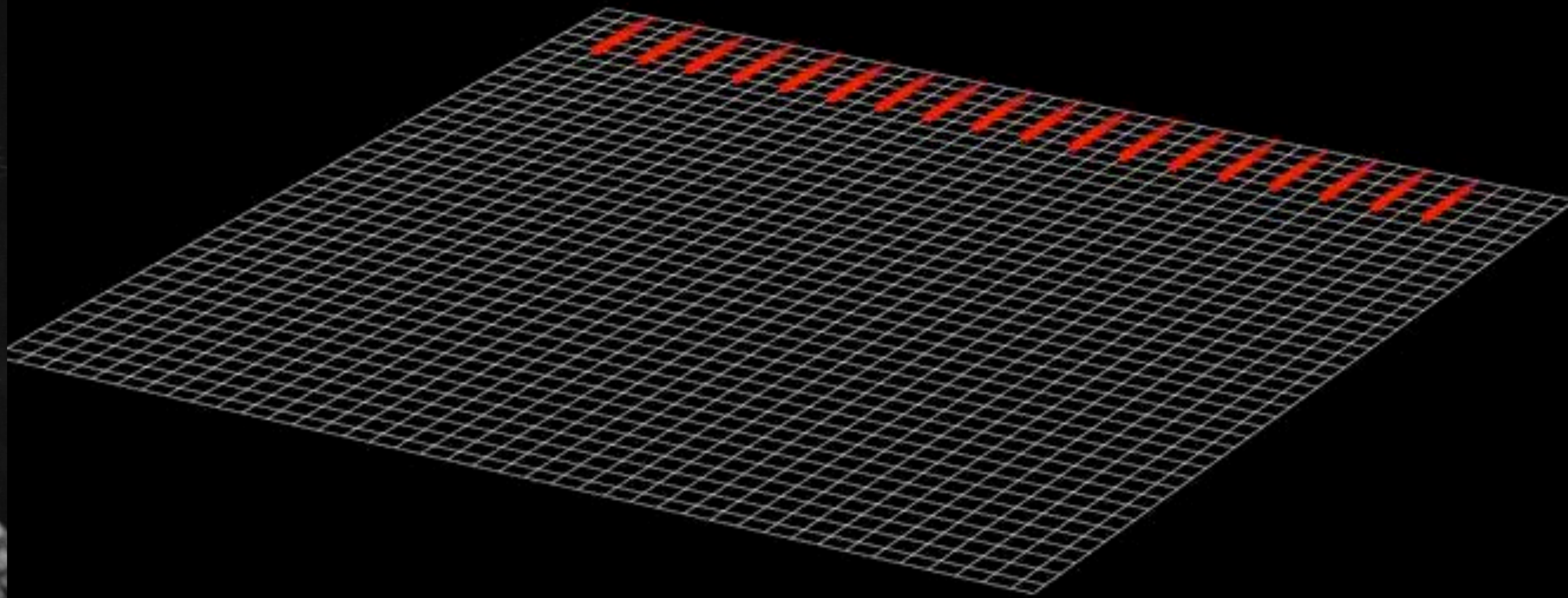
Simulating Black Hole Images





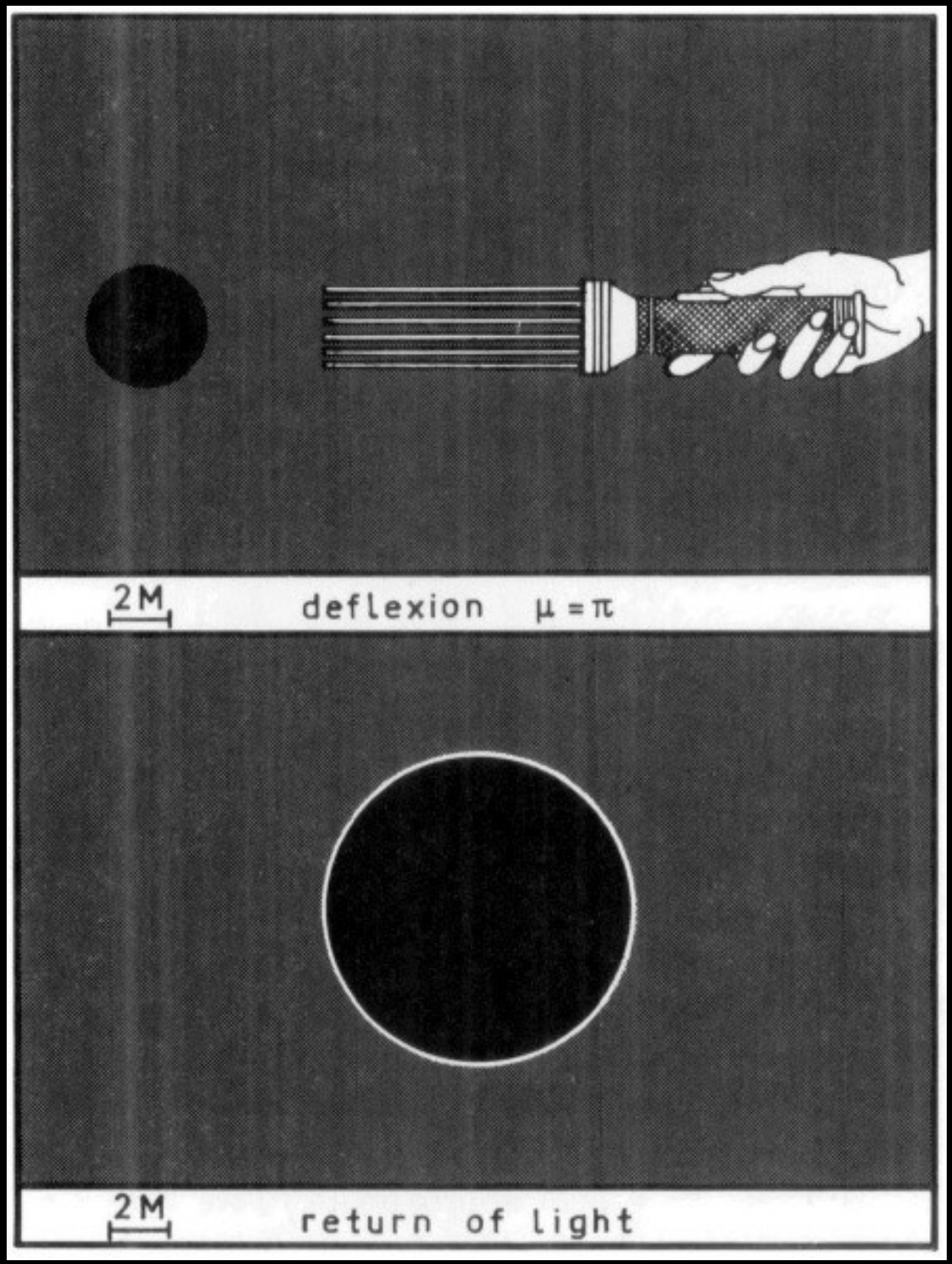
Geodesics in General Relativity

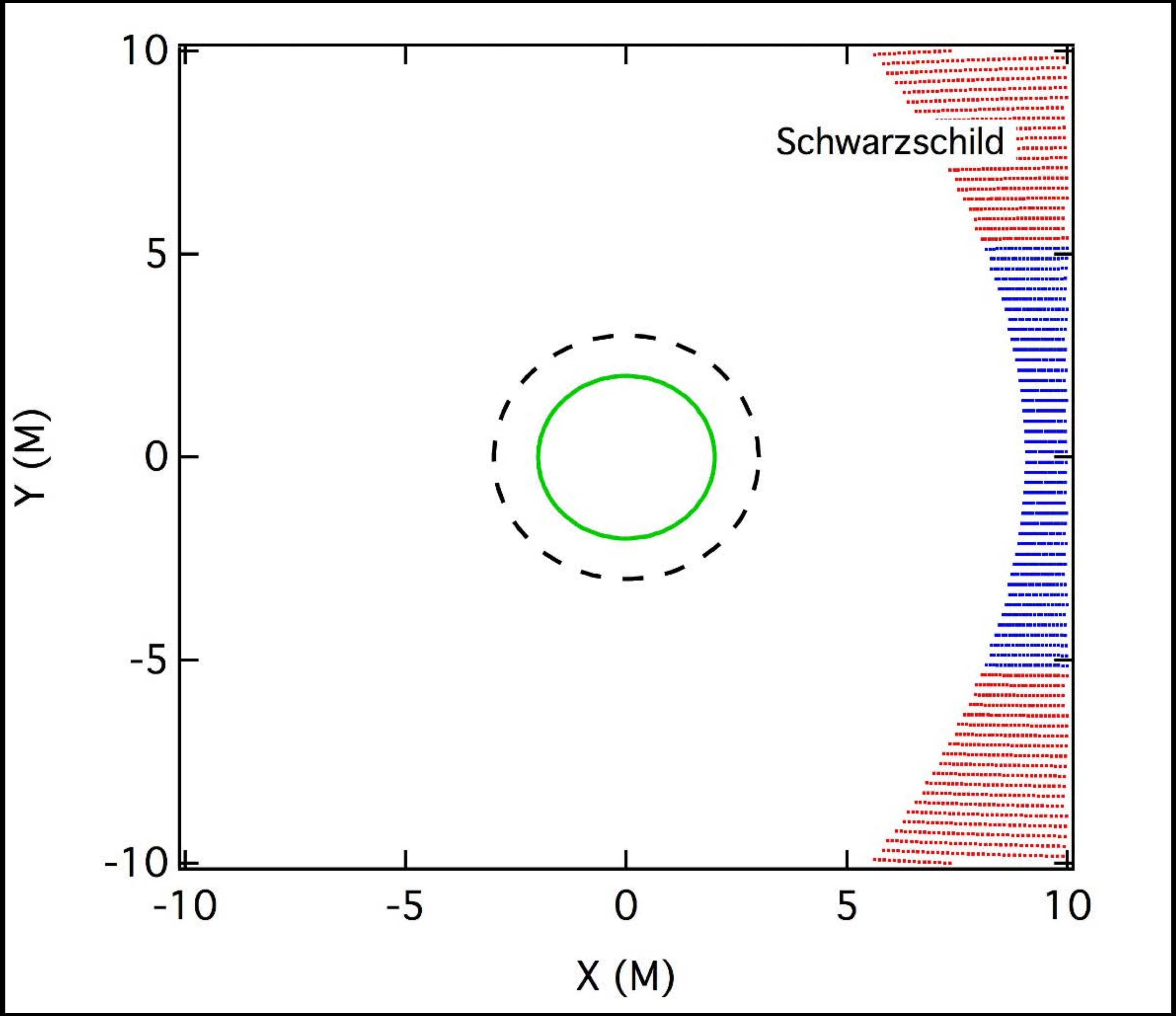




"Spacetime tells matter how to move; matter tells spacetime how to curve."
- John Wheeler

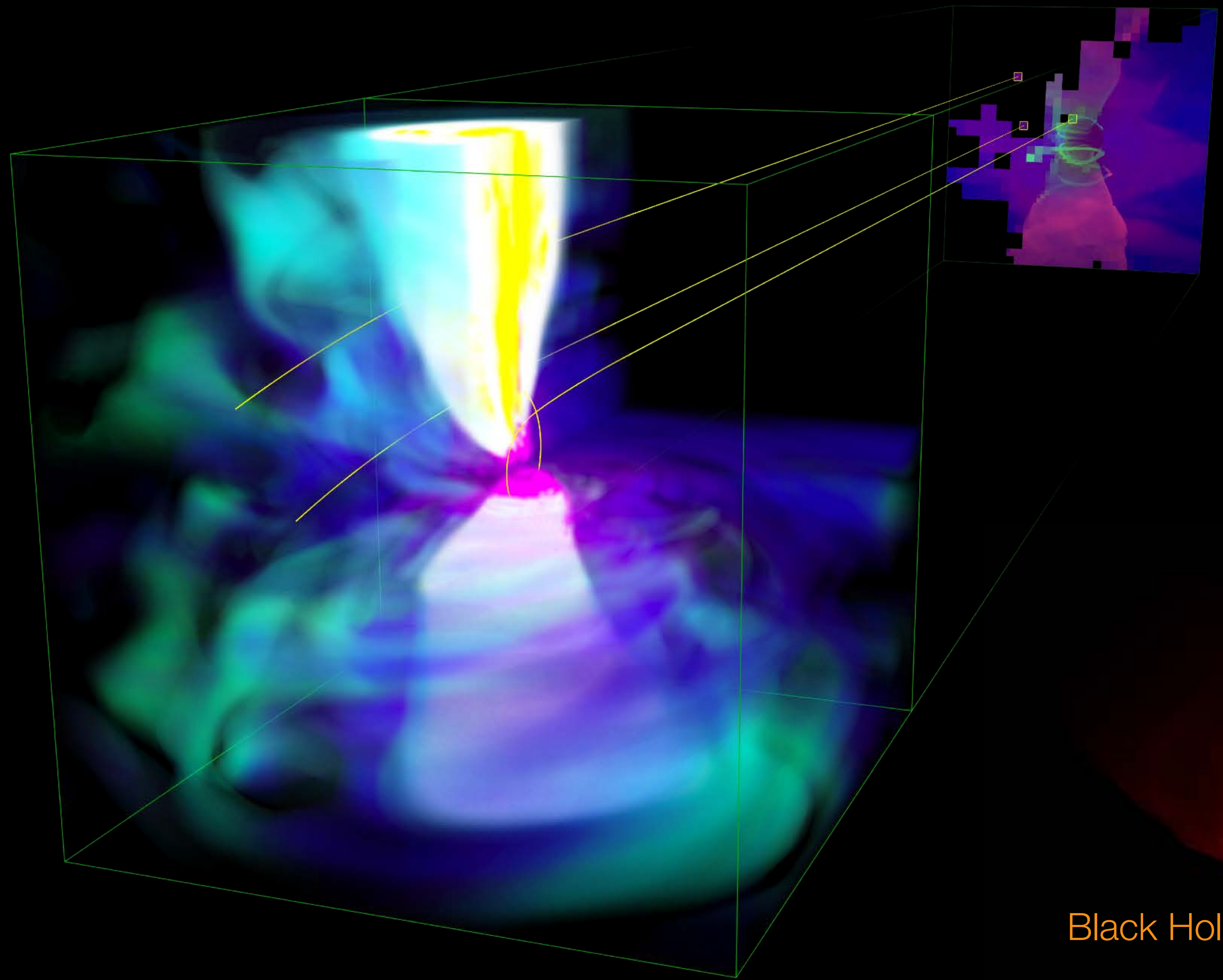
The Black Hole Shadow





Bardeen 1973, Luminet 1979

General Relativistic Ray Tracing



Black Hole PIRE

Geodesic Integration

- From action principle

$$S = \int ds = \int \sqrt{-g_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda}}d\lambda$$

- The geodesic equation is

$$\frac{d^2x^\mu}{d\lambda^2} = -\Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{d\lambda}\frac{dx^\beta}{d\lambda}$$

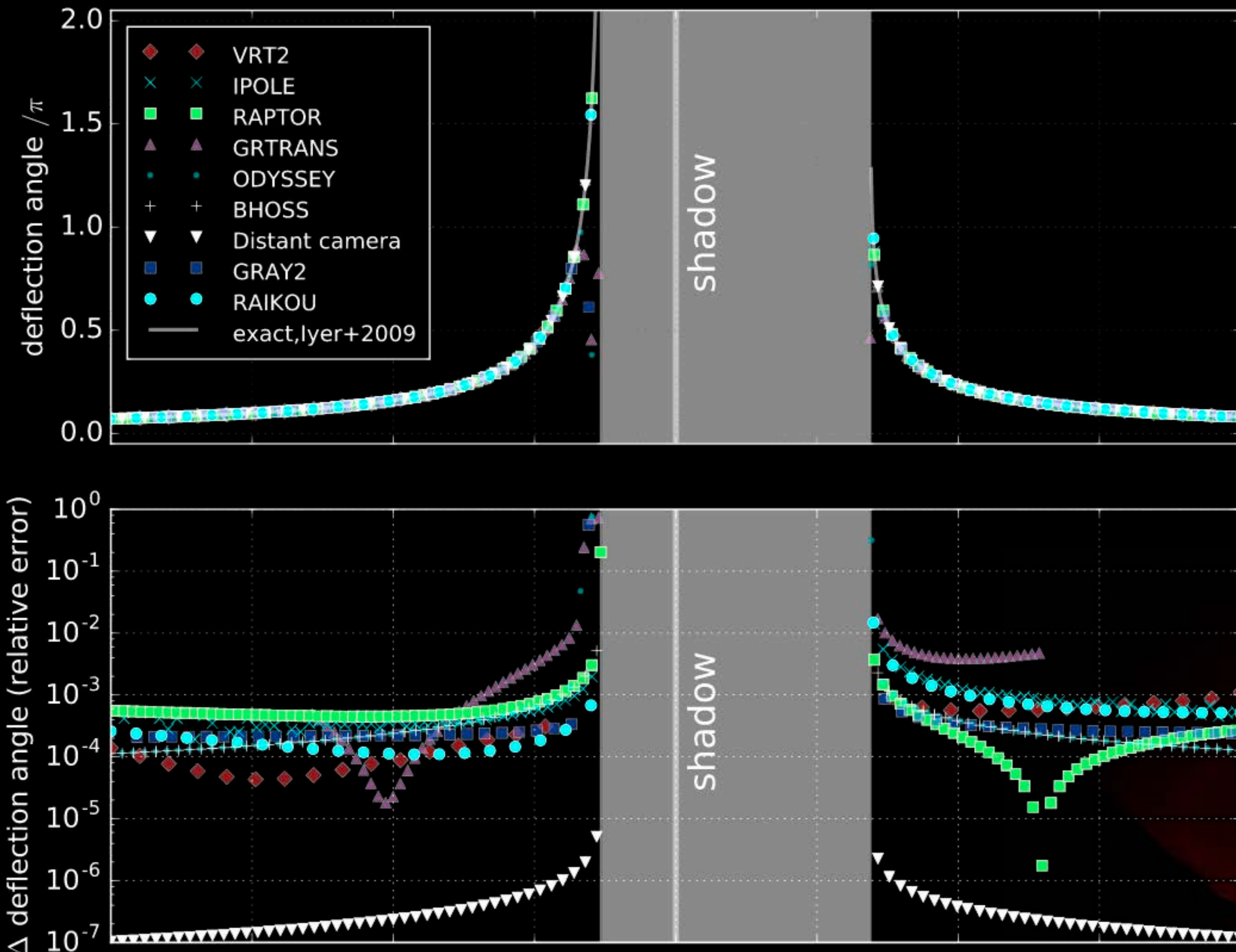
- Convenient to write it in terms of velocity

$$\frac{du^\mu}{d\lambda} = -\Gamma^\mu_{\alpha\beta}u^\alpha u^\beta$$

List of GR Ray Tracing Codes

Authors	ADS Link	Code Name	Source	Features	Numerics	Metrics
Bronzwaer, Younsi, Davelaar, & Fa	2020A&A...641A.126B	raptor2	tbronzwaer/raptor	RT+Polar		
Mościbrodzka (2020)	2020MNRAS.491.4807M	radpol		RT+Polar		
Bronzwaer, Davelaar, Younsi, Moś	2018A&A...613A...2B	raptor	tbronzwaer/raptor	RT	RK4	Arbitrary
Chan, Ozel , Psaltis, & Medeiros (	2018ApJ...867...59C	gray2	o.com/luxsrc/gray	RT	RK4	Kerr/KS
Pu, Yun, Younsi, & Yoon (2016)	2016ApJ...820..105P	odyssey	iungyipu/Odyssey	RT	RK5	Kerr/BL
Dexter (2016)	2016MNRAS.462..115D	grtrans	n/jadexter/grtrans	RT+Polar	Elliptic integrals	Kerr/BL
Chen, Kantowski, Dai, Baron, & M	2015ApJS..218....4C	kertap	/binchen14/kertap	RT	RK45	Kerr/BL
James, von Tunzelmann, Franklin	2015CQGra..32f5001J	dngr	Private	RT		Kerr/BL
Yang & Wang (2014)	2014A&A...561A.127Y	ynogkm	?J/A+A/561/A127	PT	Elliptic Integrals	Kerr/BL
Chan, Psaltis, & Ozel (2013)	2013ApJ...777...13C	gray	o.com/luxsrc/gray	RT	RK4	Kerr/BL
Schnittman & Krolik (2013)	2013ApJ...777...11S	pandurata	Private	MC+Polar	CK5	Kerr/BL
Yang & Wang (2013)	2013ApJS..207....6Y	ynogk	BCyangxl/yxl.html	RT	Elliptic integrals	Kerr/BL
Baubock, Psaltis, Ozel, & Johann	2012ApJ...753..175B	ray	Private	RT	RK4	(Kerr+Qpole)/BL
Psaltis & Johannsen (2012)	2012ApJ...745....1P	ray	Private			QuasiKerr/BL
Shcherbakov & Huang (2011)	2011MNRAS.410.1052S	astroray	code/ASTRORAY/	RT+Polar	RK2	Kerr/BL
Vincent, Paumard, Gourgoulhon, (	2011CQGra..28v5011V	gyoto	//gyoto.obspm.fr/	RT		Kerr/BL + Numeric
Dolence, Gammie, Moscibrodzka	2009ApJS..184..387D	grmonty	nois.edu/codelib/	MC	RK4	Kerr/BL
Dexter & Agol (2009)	2009ApJ...696.1616D	geokerr	okerr/index.html	RT	Elliptic integrals	Kerr/BL
Schnittman (2005)	2005PhDT........24S		Private	MC		Kerr/BL
Beckwith & Done (2005)	2005MNRAS.359.1217B		Private			Kerr/BL
...						
Luminet (1979)	1979A&A....75..228L		Private			
Cunningham & Bardeen (1973)	1973ApJ...183..237C		Private			Kerr/BL

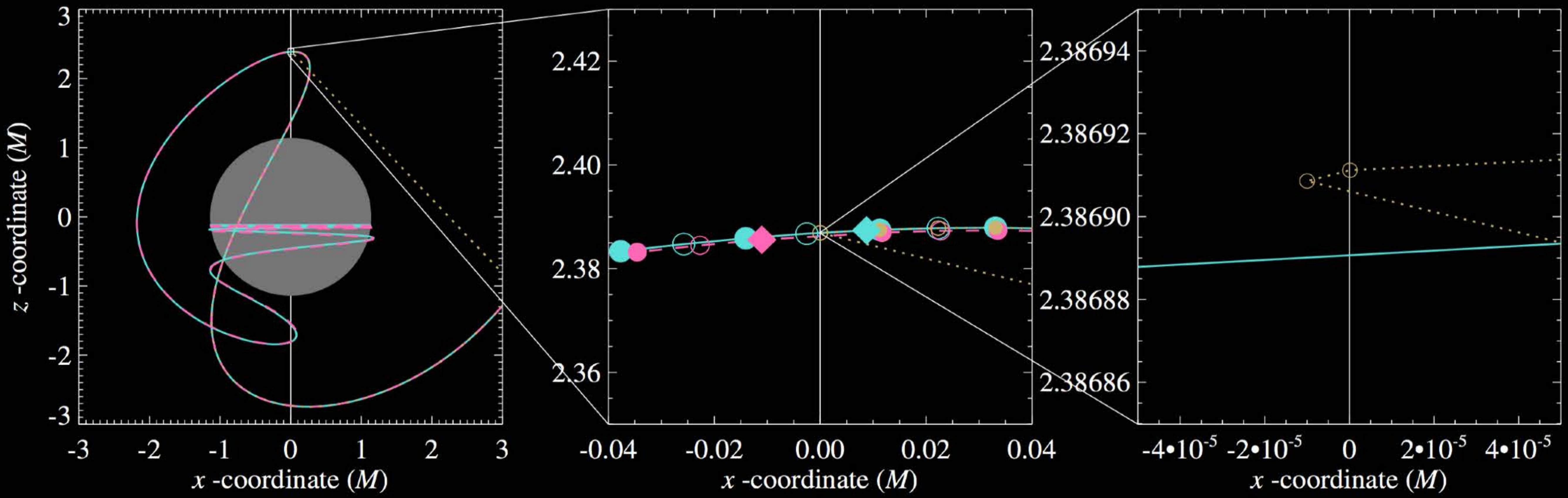
GRRT Code Comparison



Gold, et al. EHTC (2020)

Pole Treatment

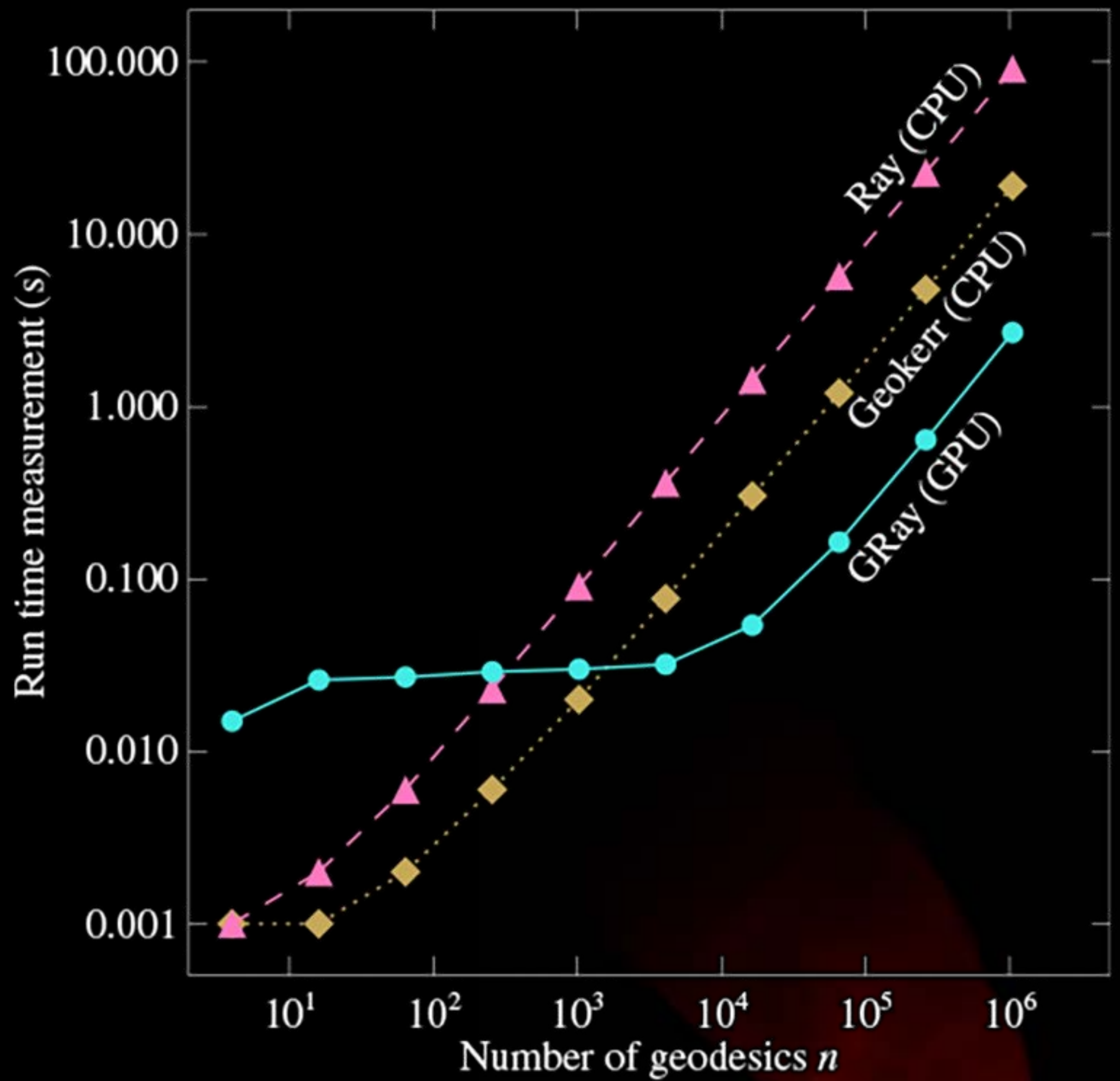
- Boyer-Lindquist has coordinate singularity



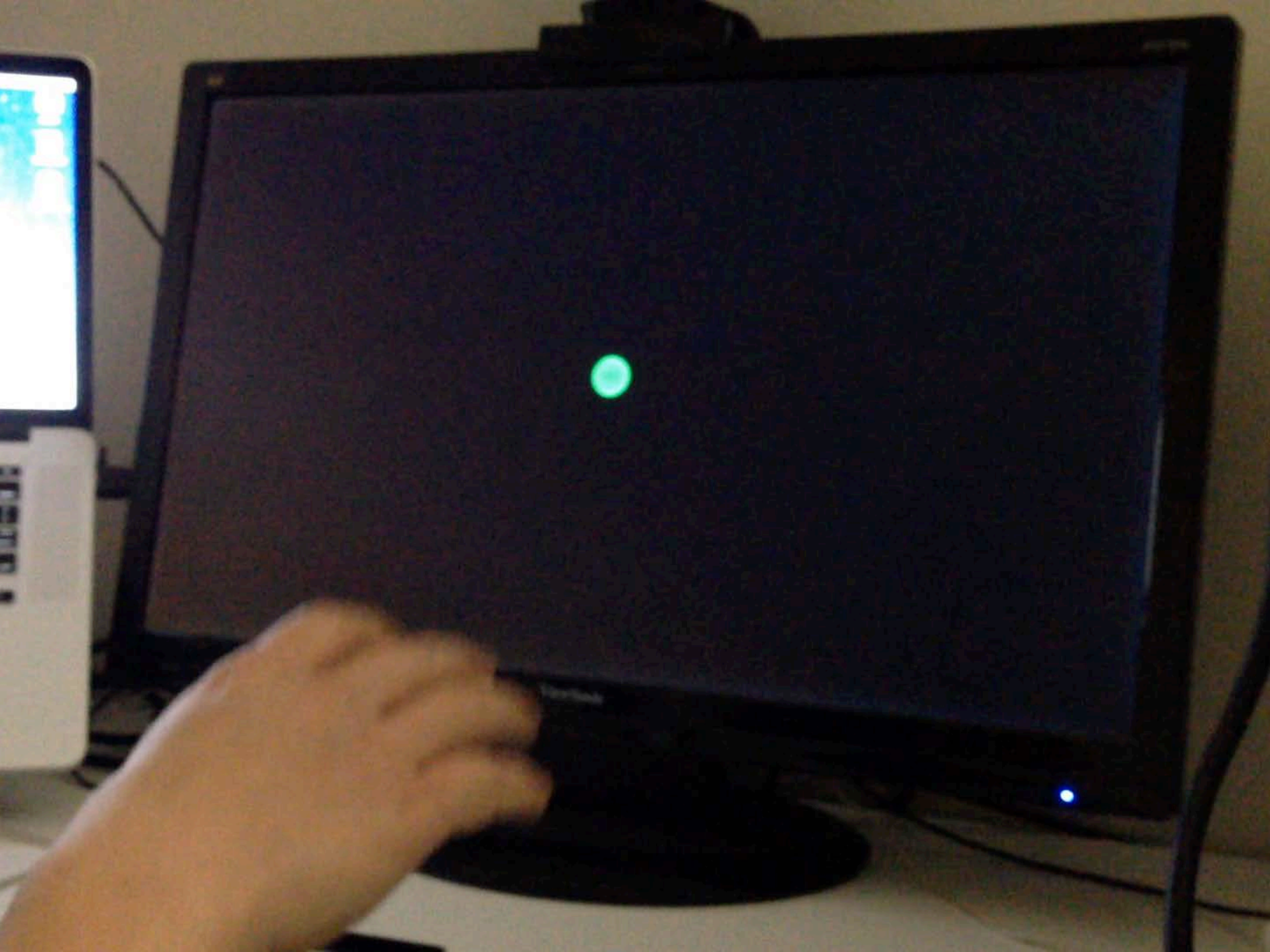
- Fall back to forward Euler near the poles

GPU Acceleration

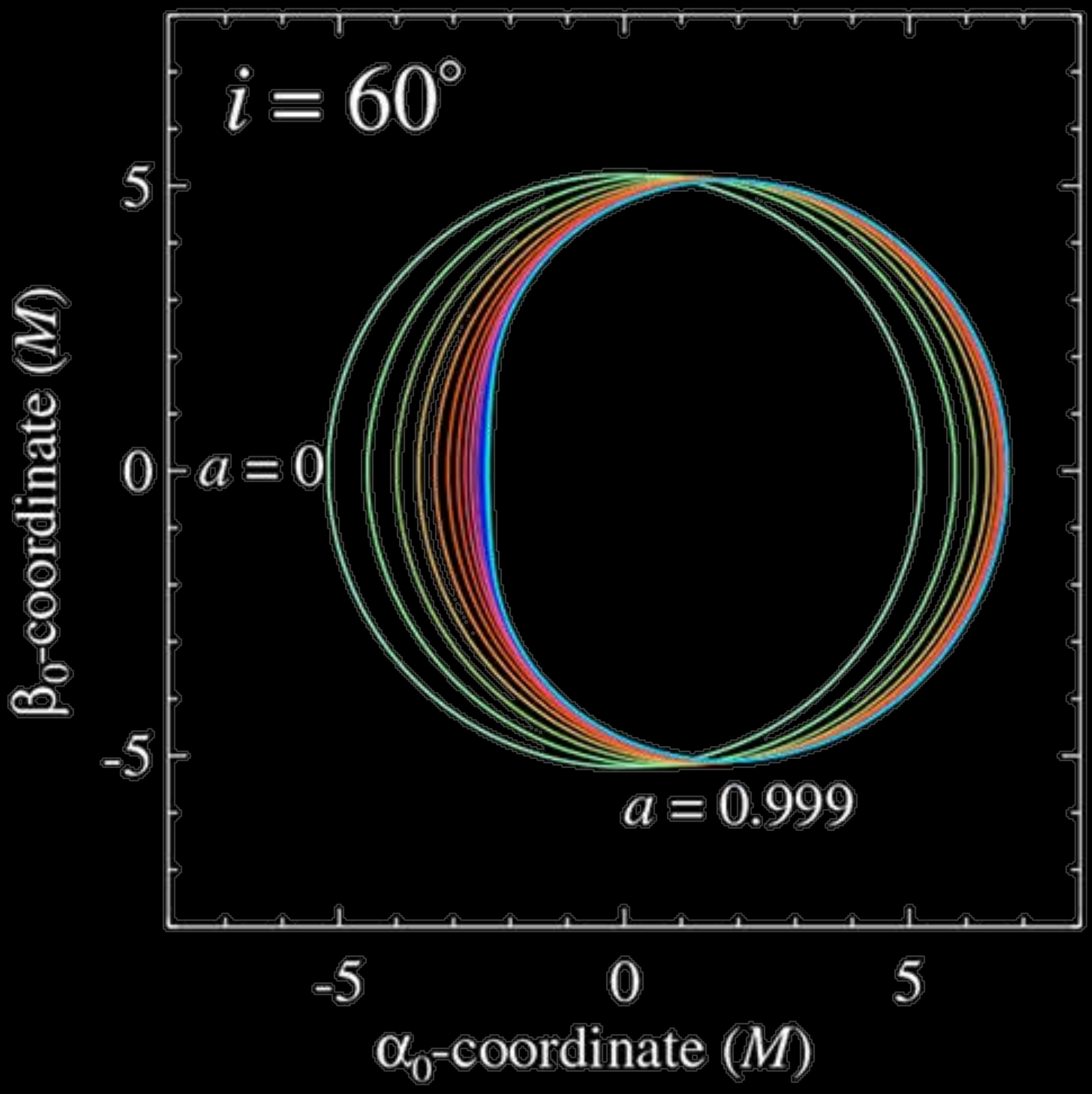
- First general relativistic ray tracing code on GPUs
- 30x faster than CPU codes using same numerical method
- Open source (GPLv3) and available on GitHub https://github.com/luxsrc/gray
- Fast enough to perform interactive ray tracing
- Makes great movies!

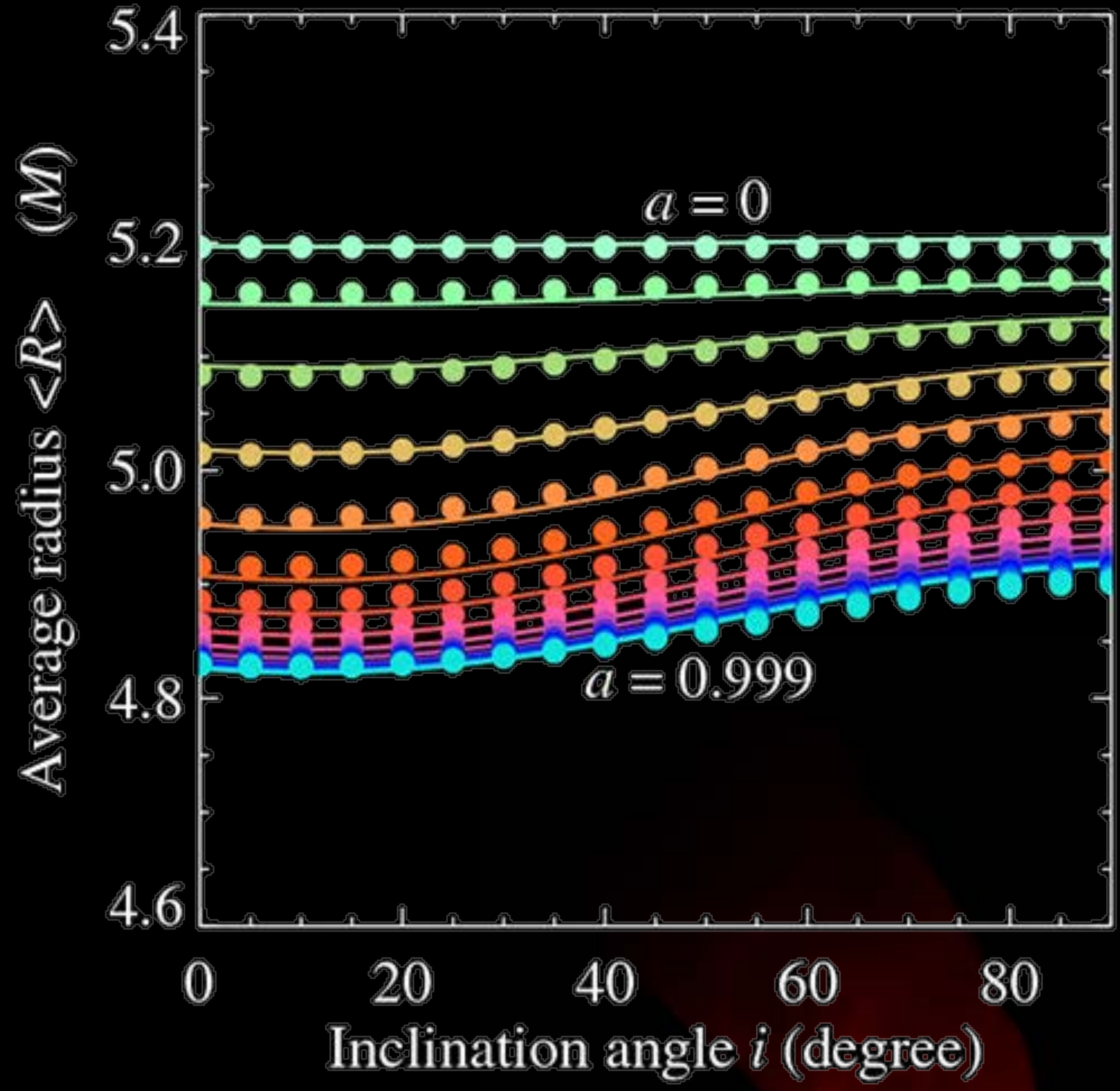


Black Hole PIRE



Signature of General Relativity





Radiative Transfer

- Non-relativistic radiative transfer equation:

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu \qquad\qquad \frac{dI_\nu}{d\tau_\nu} = -I_\nu + \frac{j_\nu}{\alpha_\nu}$$

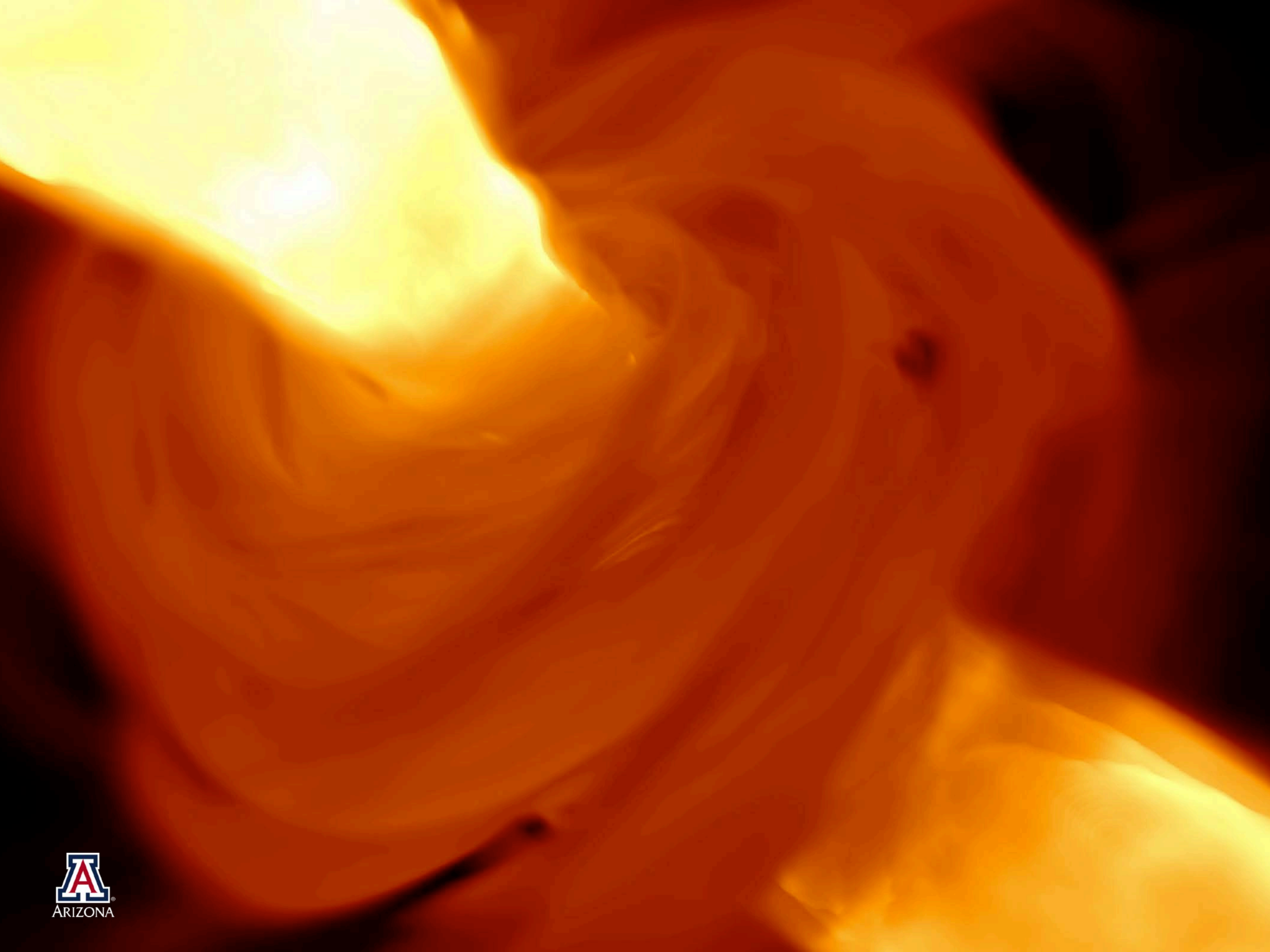
- Turns out that $\mathcal{I} \equiv I_\nu/\nu^3$, $\chi = \nu\alpha_\nu$, and $\eta \equiv j_\nu/\nu^2$ Lorentz-invariant. Let τ_ν be the optical depth, the radiative transfer equation can be written as

$$\frac{d\mathcal{I}}{d\tau_\nu} = -\mathcal{I} + \frac{\eta}{\chi}$$

- Using integration factor,

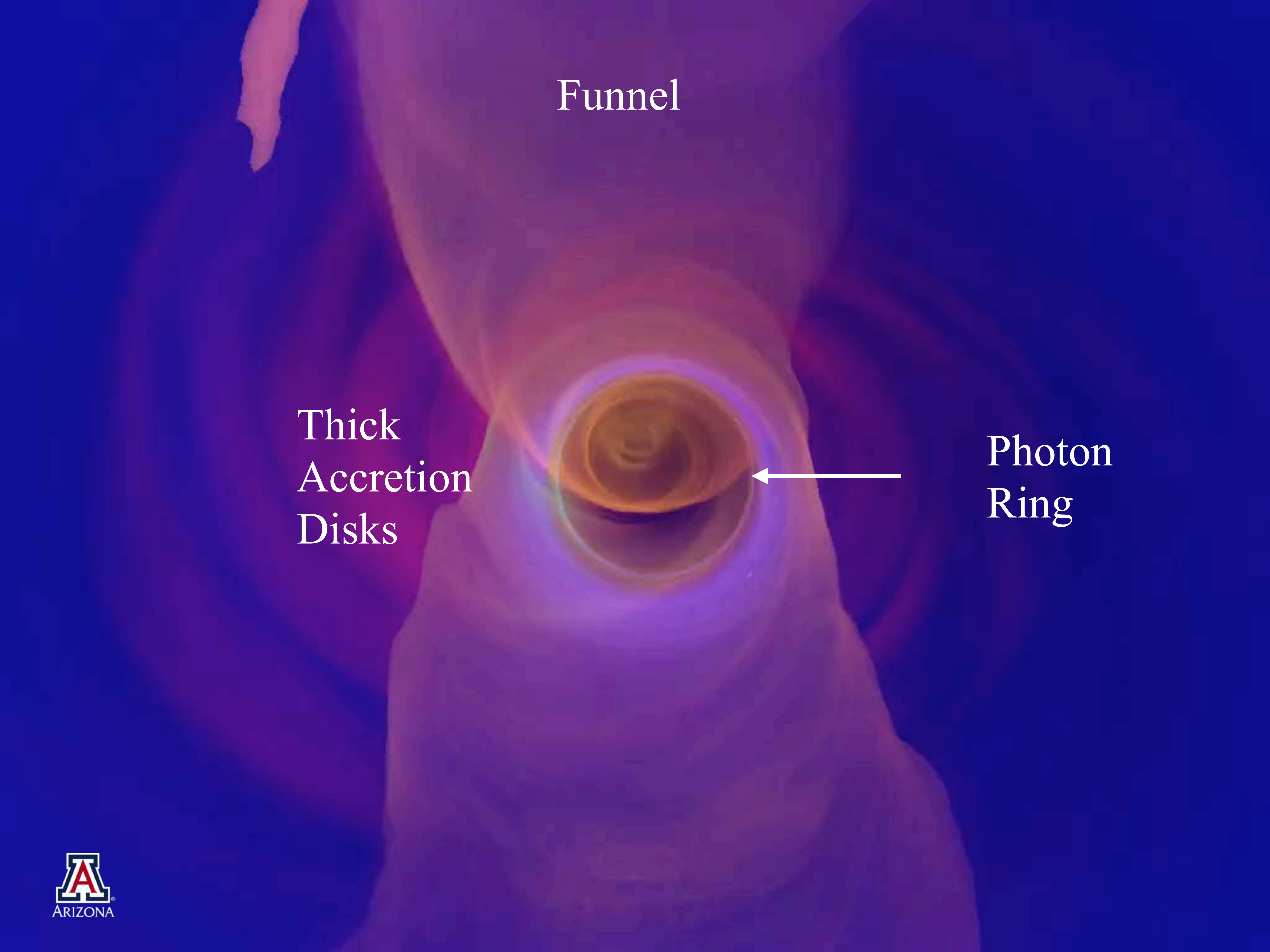
$$\frac{d\tau_\nu}{d\lambda} = \gamma^{-1}\alpha_{0,\nu}$$

$$\frac{d\mathcal{I}}{d\lambda} = \gamma^{-1}\left(\frac{j_{0,\nu}}{\nu^3}\right)e^{-\tau_\nu}$$


ARIZONA

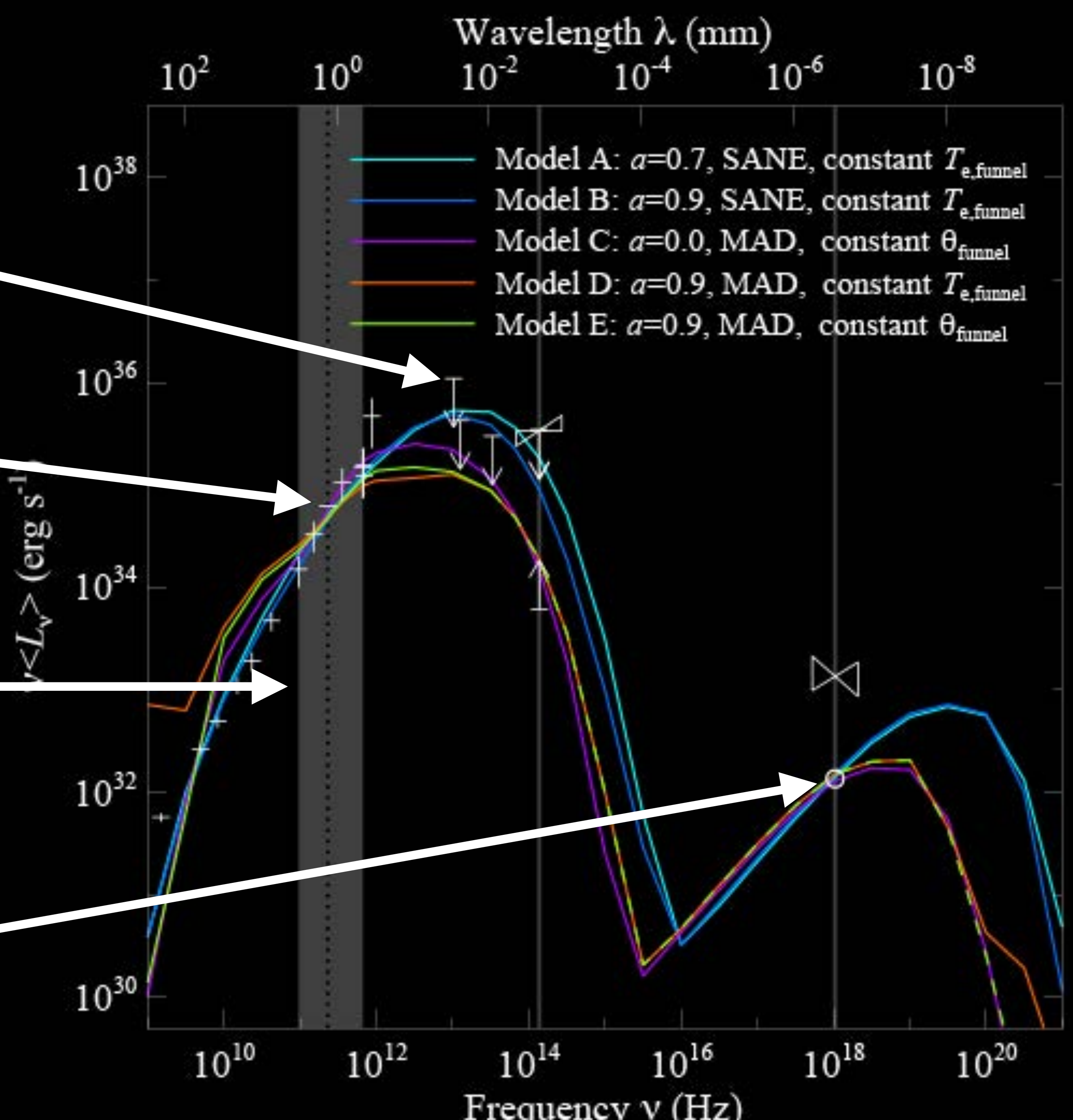
Single Precision Radiative Transfer

```
static inline __device__ real
B_Planck(real nu, real te)
{
  real f1 = 2 * CONST_h * CONST_c;          // ~4e-16
  real f2 = CONST_h / (CONST_me * CONST_c); // ~2e-10
  nu /= K(CONST_c);               // 1e-02--1e+12
  f1 *= nu * nu;                  // 4e-20--4e+08
  f2 *= nu / (te + EPSILON);      // 1e-12--1e+02
  return nu * (f2 > K(1e-5) ?
               f1 / (EXP(f2) - 1) :
               f1 / f2 / (1 + f2/2 + f2*f2/6));
} // 10+ FLOP
```


Funnel
Thick
Accretion
Disks
Photon
Ring
ARIZONA

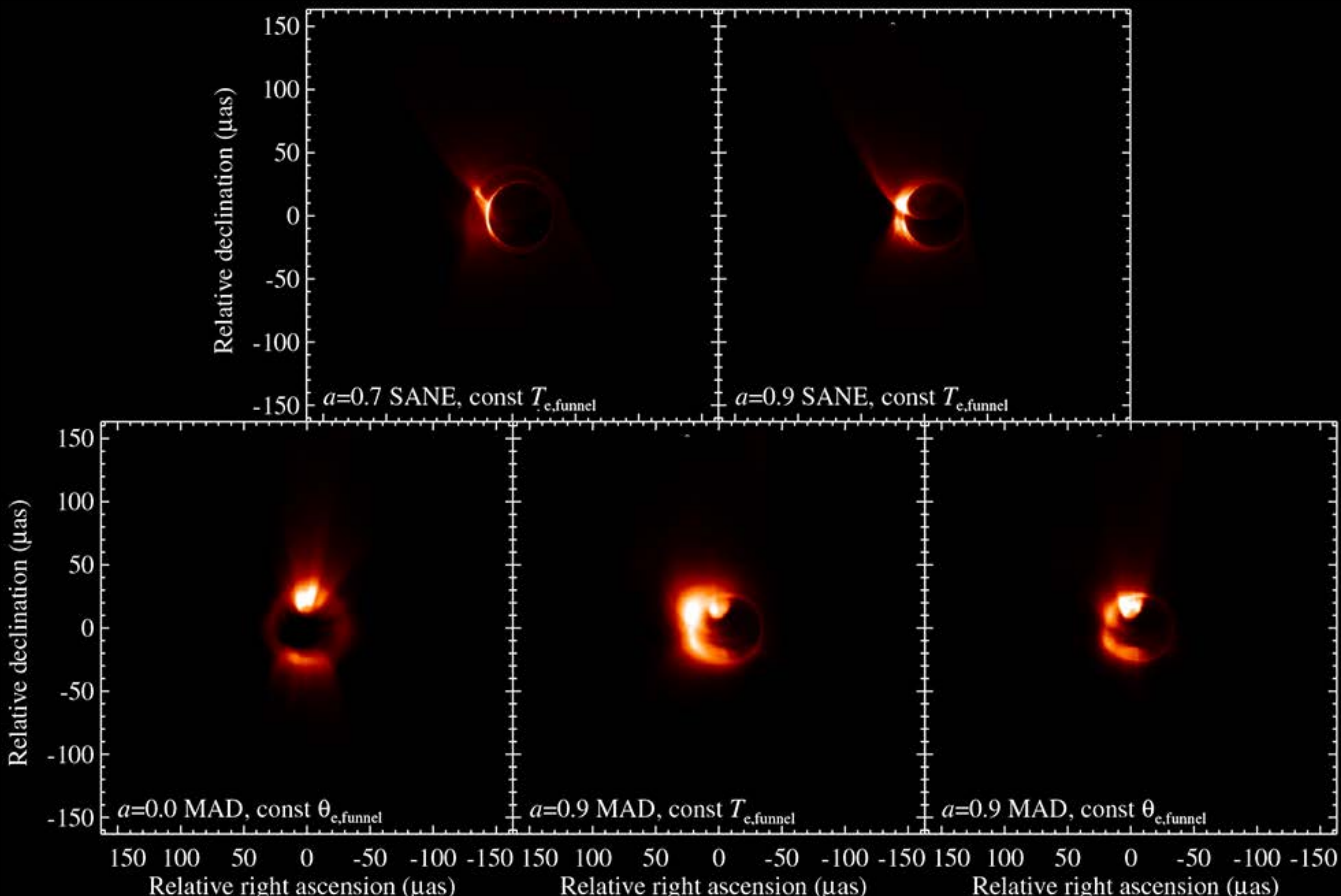
Fit Model to Data

4. Impose upper and lower bounds
3. Image size at 1.3mm from early EHT observations
2. Fit to flux values within grey band
1. Fix density by fitting to 10% of quiescent X-ray

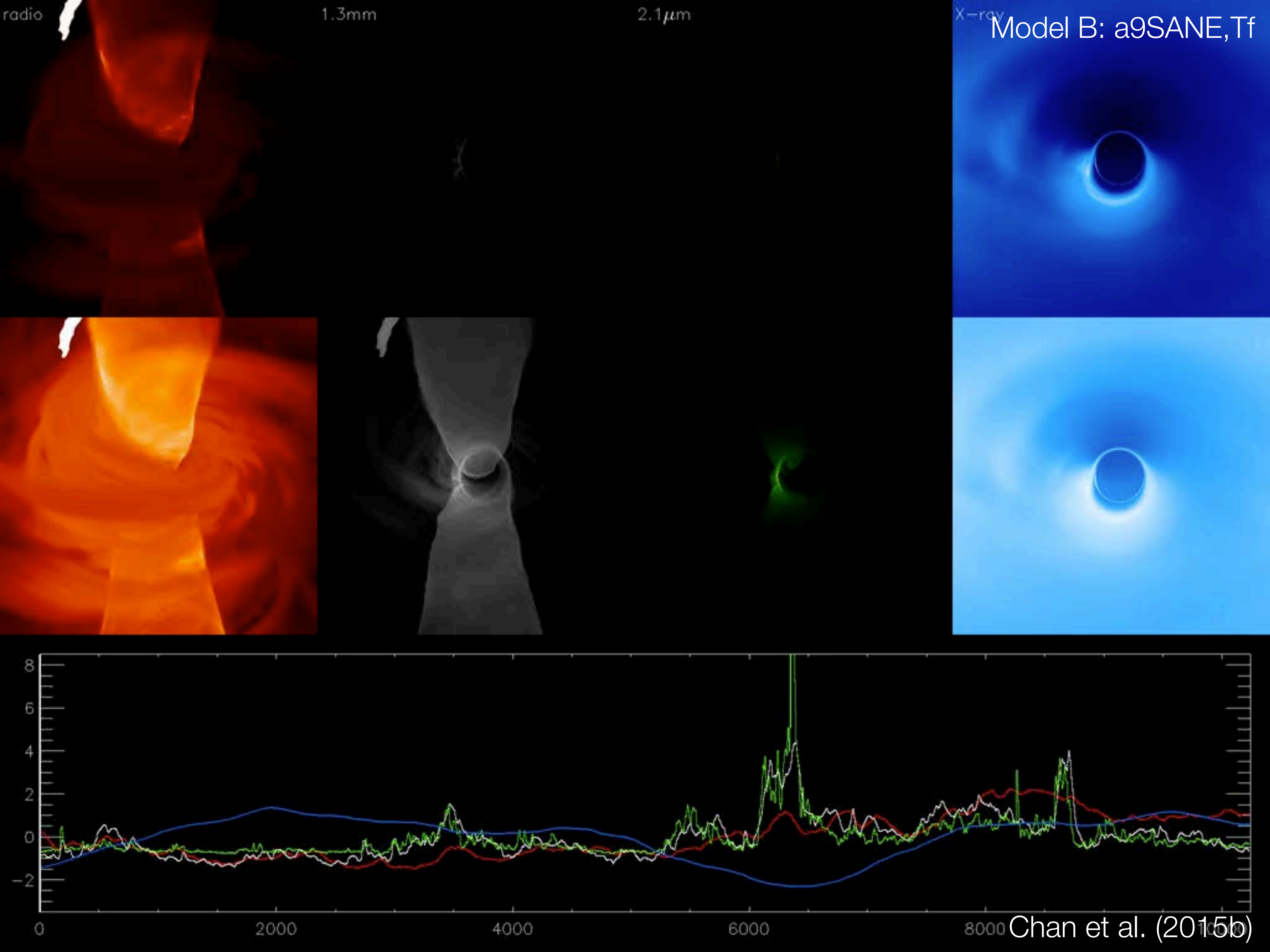


Chan et al. (2015a)

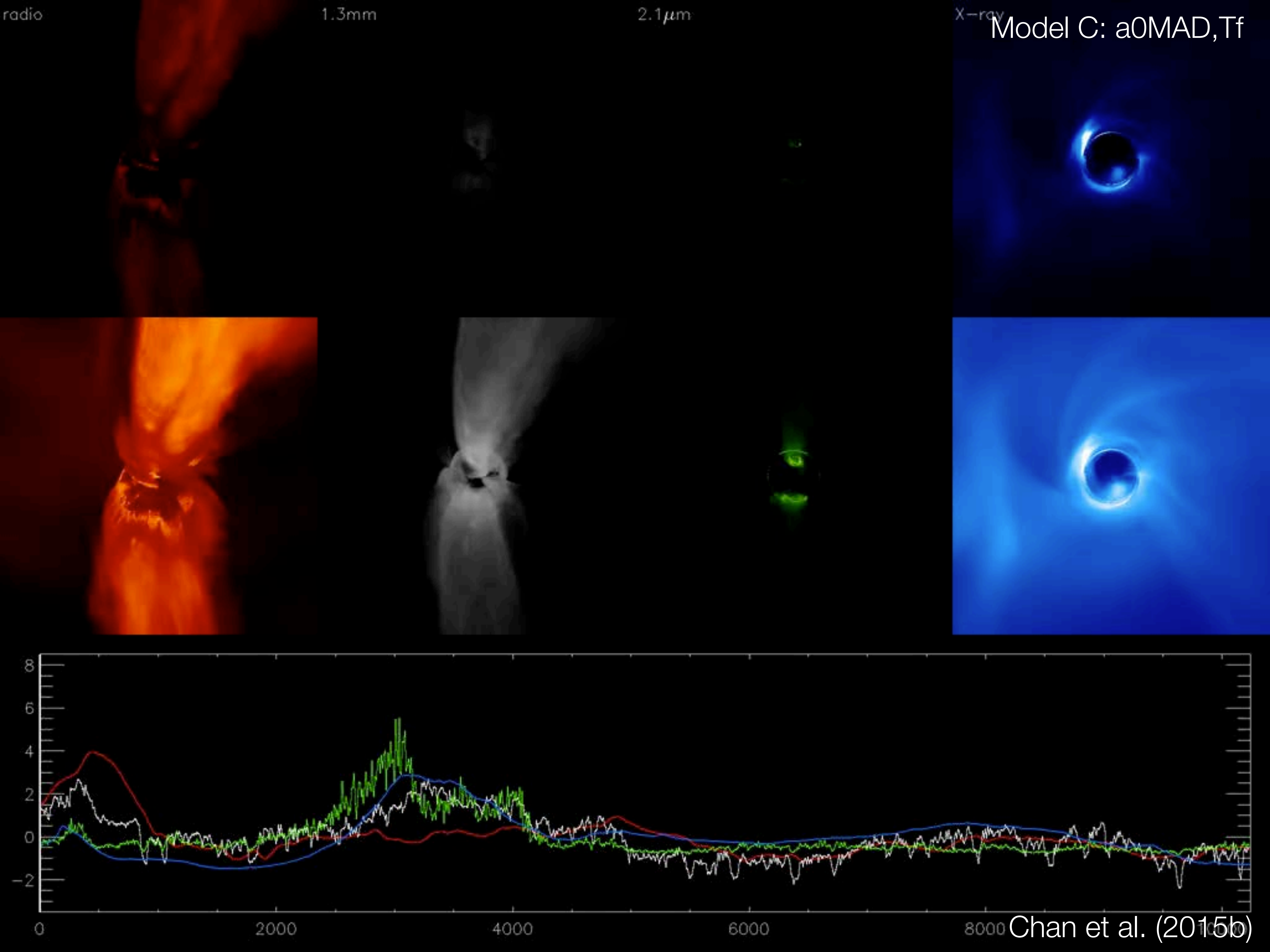
Multiple Best Models

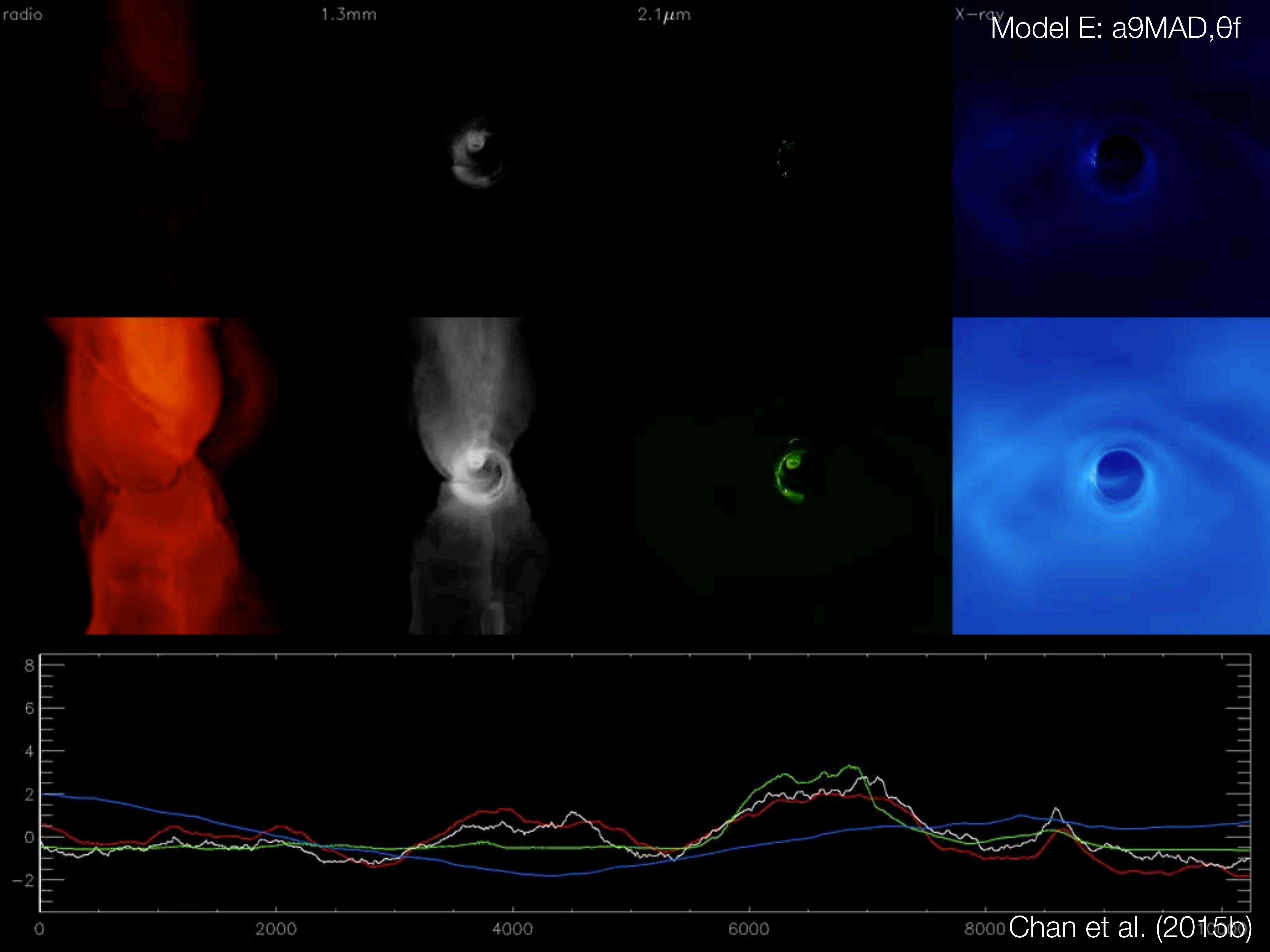


Chan et al. (2015a)



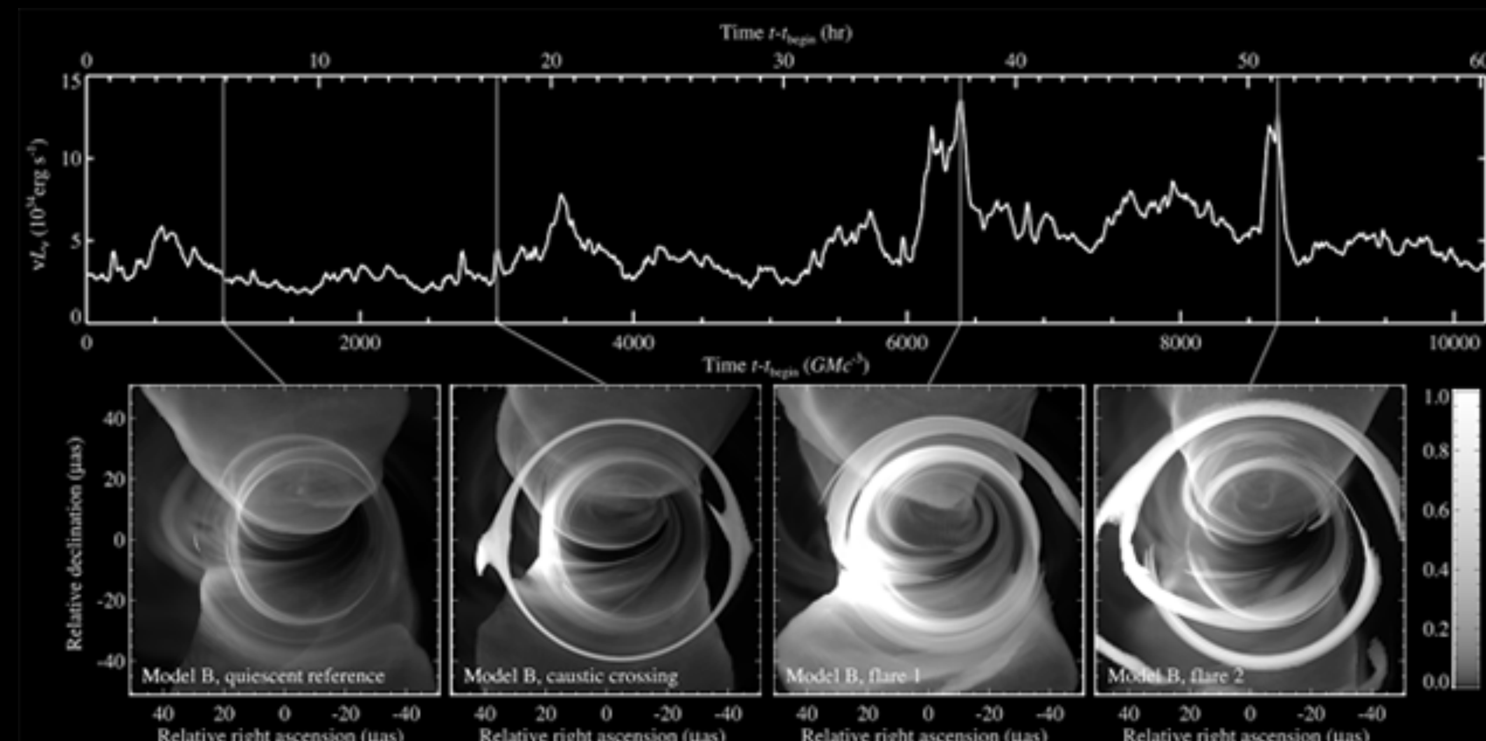
radio
1.3mm
2.1μm
X-ray
Model B: a9SANE,Tf
8
6
4
2
0
−2
0
2000
4000
6000
8000
Chan et al. (2015b)


radio
1.3mm
2.1μm
X-ray
Model C: a0MAD,Tf
8
6
4
2
0
−2
0
2000
4000
6000
8000
Chan et al. (2015b)


radio
1.3mm
2.1μm
X-ray
Model E: a9MAD,θf
8
6
4
2
0
−2
0
2000
4000
6000
8000
Chan et al. (2015b)

What Causes the Variability?

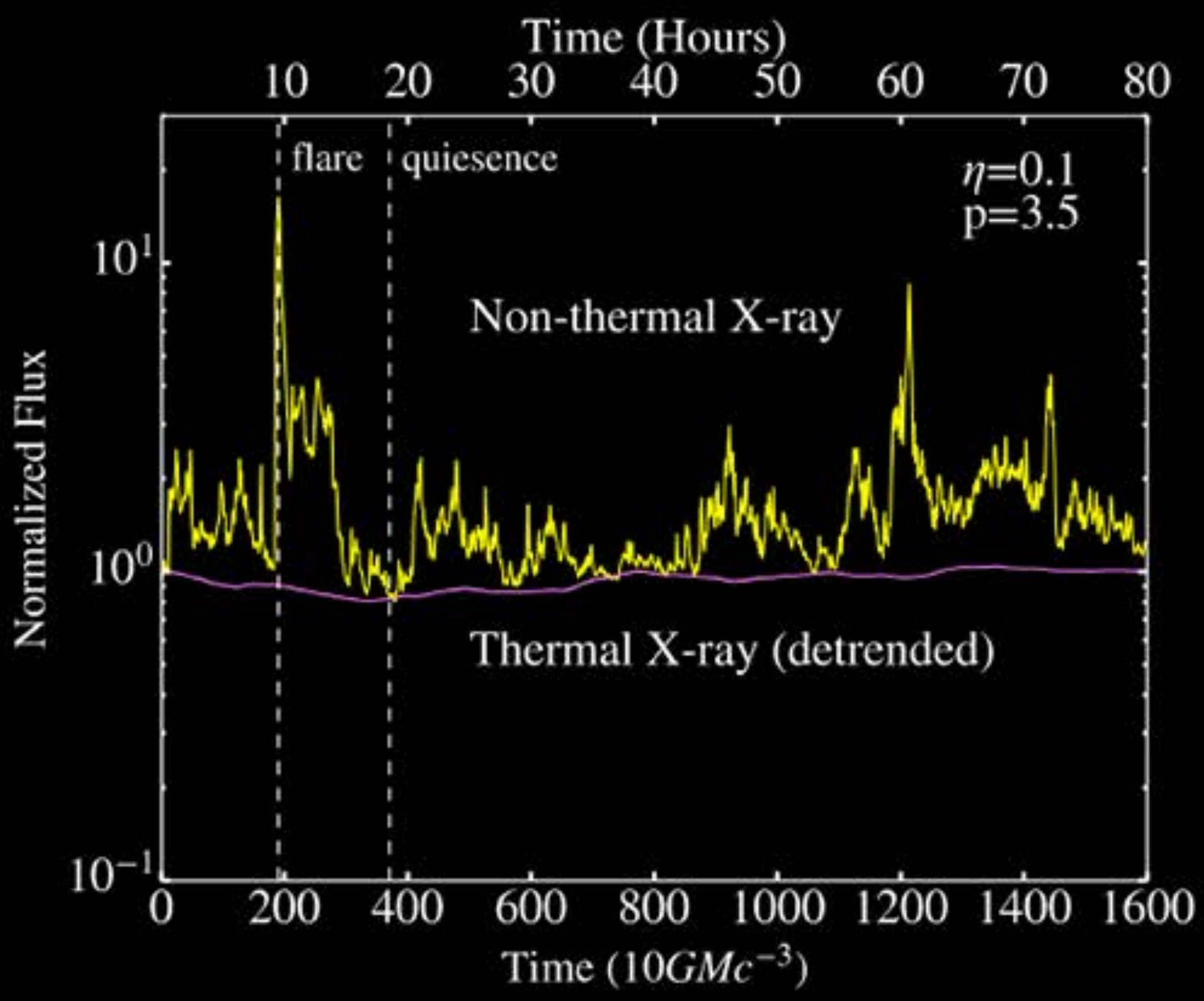
- Origins of variability:
    - Turbulence fluctuations
    - Magnetic Reconnections
    - Strong magnetic flux tubes

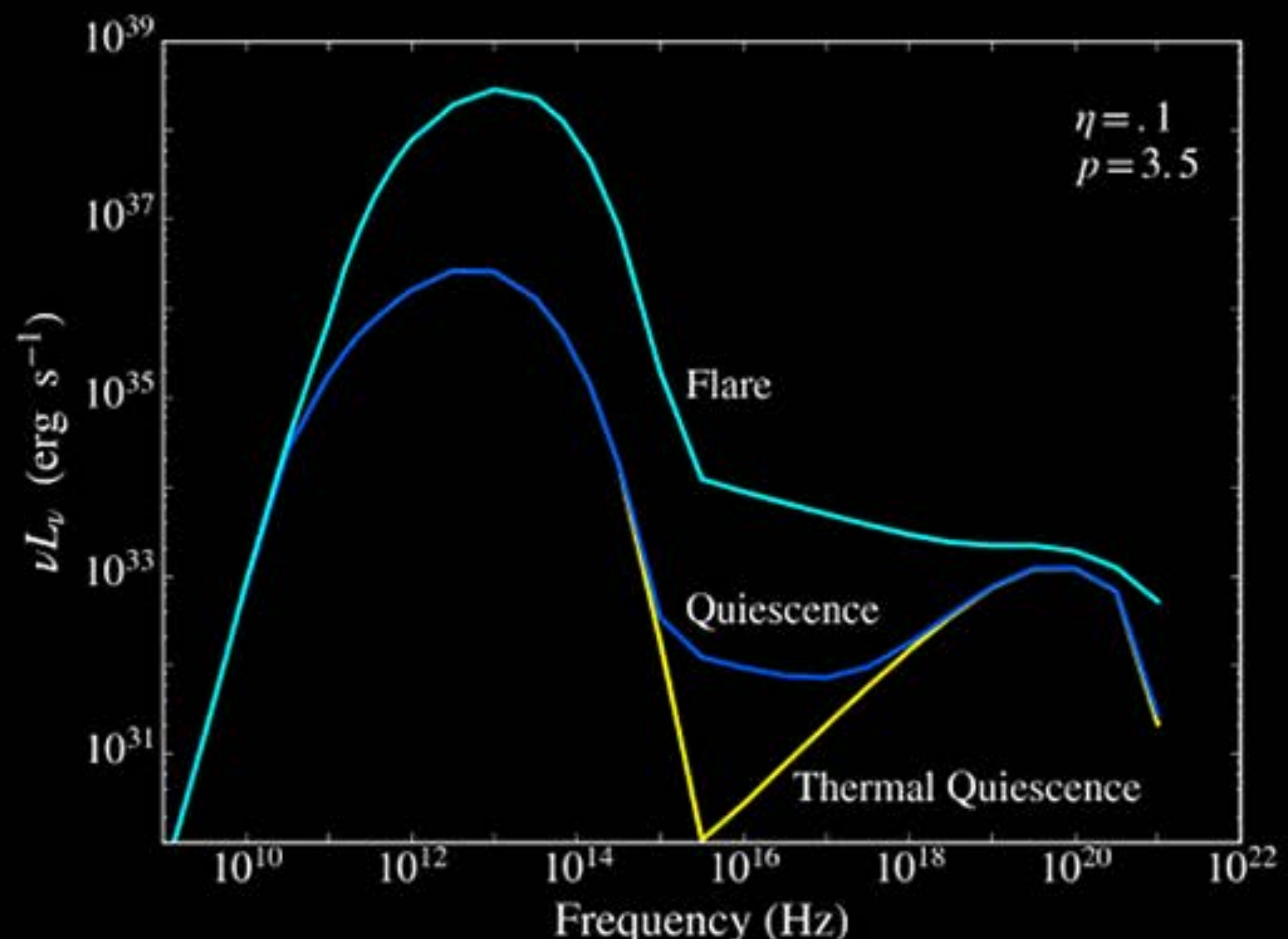


Chan et al. (2015b)

- Strong gravitational lensing creates observable features but does not change the flux too much
- No X-ray flare if non-thermal electrons are ignored

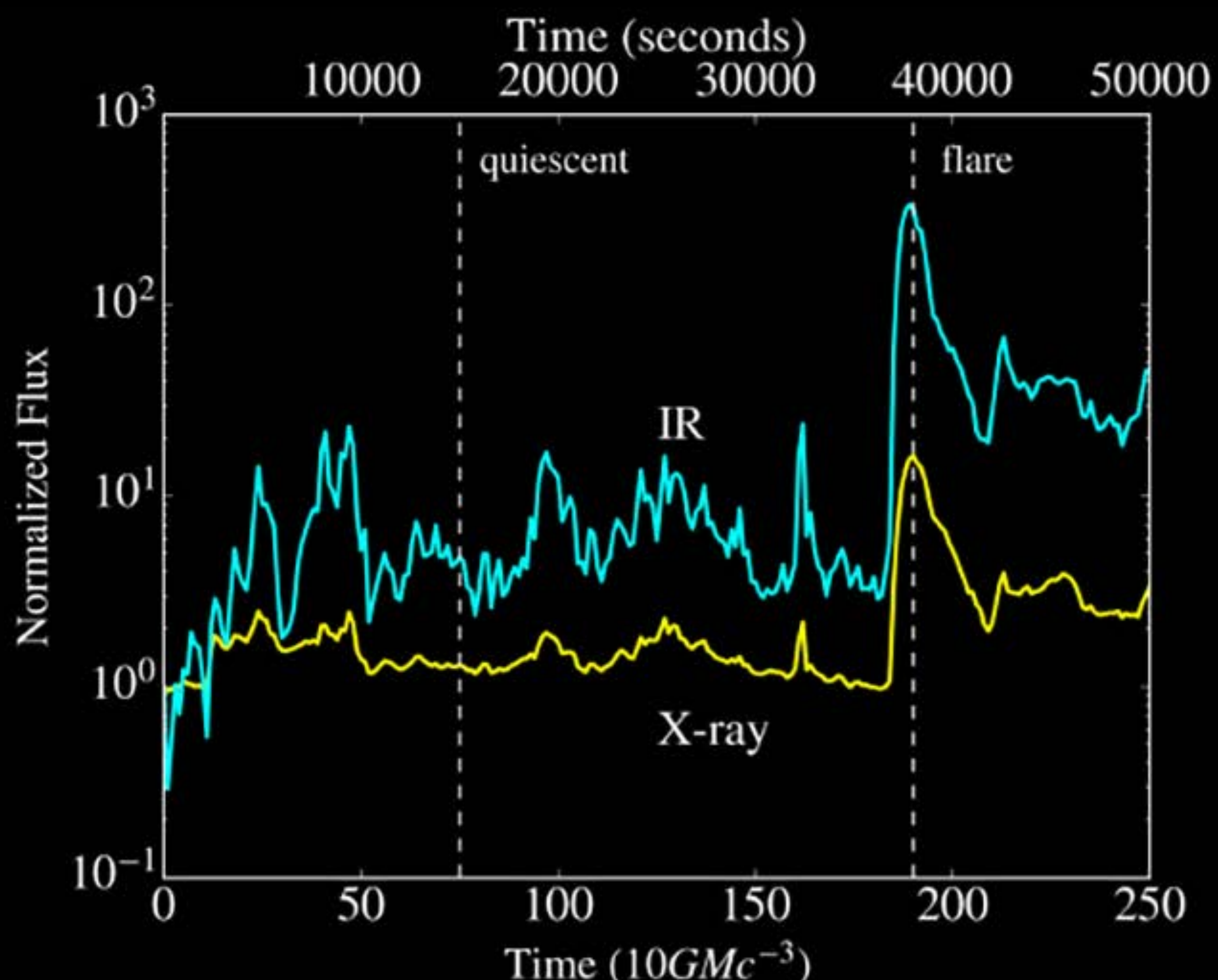
Adding Non-thermal Electrons





Ball, Ozel, Psaltis, & Chan (2015)

Adding Non-thermal Electrons



Ball, Ozel, Psaltis, & Chan (2015)

Limitations

- "The theory of high temperature, collisionless plasmas must be better understood if this core physical uncertainty of sub-Eddington black hole accretion is to be eliminated" EHTC M87 Paper V (2019)
  - Electron-ion temperature ratio
  - Electron acceleration mechanisms
  - Electron distribution function (eDF)
- Low density regions in the funnel/jet
  - Choice of density floor
  - Emission cutoff from region with $B^2/\rho > 1$

Recent Work and Related Topics

- Code optimization beyond CUDA
- Better handle of coordinate singularities

} GRay2

- Geodesic solver for non-Kerr metrics
- Long term integration (especially for particles)

} FANTASY

- Polarized radiative transfer
- Scattering in radiative transfer
- Physical speed of light
- Radiation hydrodynamics
- ...

GRay2

- Most GR ray tracing codes use Boyer-Lindquist coordinates

$$
\begin{aligned}
ds^2 = &- \frac{\Delta}{\Sigma}(dt - a\sin^2\theta\, d\phi)^2 \\
&+ \frac{\sin^2\theta}{\Sigma}\Big[(r^2 + a^2)d\phi - a dt\Big]^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2
\end{aligned}
$$

- Probably due to the fact there is only one (independent) off-diagonal element in the metric
- However, the spherical-polar nature of these coordinates have singularities along the poles

GRay2

- In GRay2, we use the Kerr–Schild "Cartesian" coordinates

$$g_{\mu\nu} = \eta_{\mu\nu} + f l_\mu l_\nu$$

$$f \equiv \frac{2r^3}{r^4 + a^2 z^2} \qquad l_\mu = \left(1, \frac{rx + ay}{r^2 + a^2}, \frac{ry - ax}{r^2 + a^2}, \frac{z}{r}\right)$$

$$x^2 + y^2 + z^2 = r^2 + a^2(1 - z^2/r^2)$$

- No coordinate singularity; no assumption on particle velocity
- HARM uses Kerr-Schild spherical-polar coordinates: trivial coordinate transforms between GRay2 and HARM

GRay2

- Kerr–Schild Cartesian metric does not have any zero element
- Computing all the Christoffel symbols seems expensive
- But we can simplify the geodesic equations:

$$\begin{aligned}\frac{du^\mu}{d\lambda} &= -\frac{1}{2}g^{\mu\nu}(g_{\nu\alpha,\beta} + g_{\nu\beta,\alpha} - g_{\alpha\beta,\nu})u^\alpha u^\beta \\ &= -g^{\mu\nu}g_{\nu\alpha,\beta}u^\alpha u^\beta + \frac{1}{2}g^{\mu\nu}g_{\alpha\beta,\nu}u^\alpha u^\beta \\ &= -g^{\mu\beta}g_{\beta\gamma,\alpha}u^\gamma u^\alpha + \frac{1}{2}g^{\mu\alpha}g_{\beta\gamma,\alpha}u^\beta u^\gamma \\ &= -\left(g^{\mu\beta}u^\alpha - \frac{1}{2}g^{\mu\alpha}u^\beta\right)g_{\beta\gamma,\alpha}u^\gamma\end{aligned}$$

The complicated part is only second rank

GRay2

- Putting in the Kerr–Schild "Cartesian" metric
- The geodesic equations are not that bad!

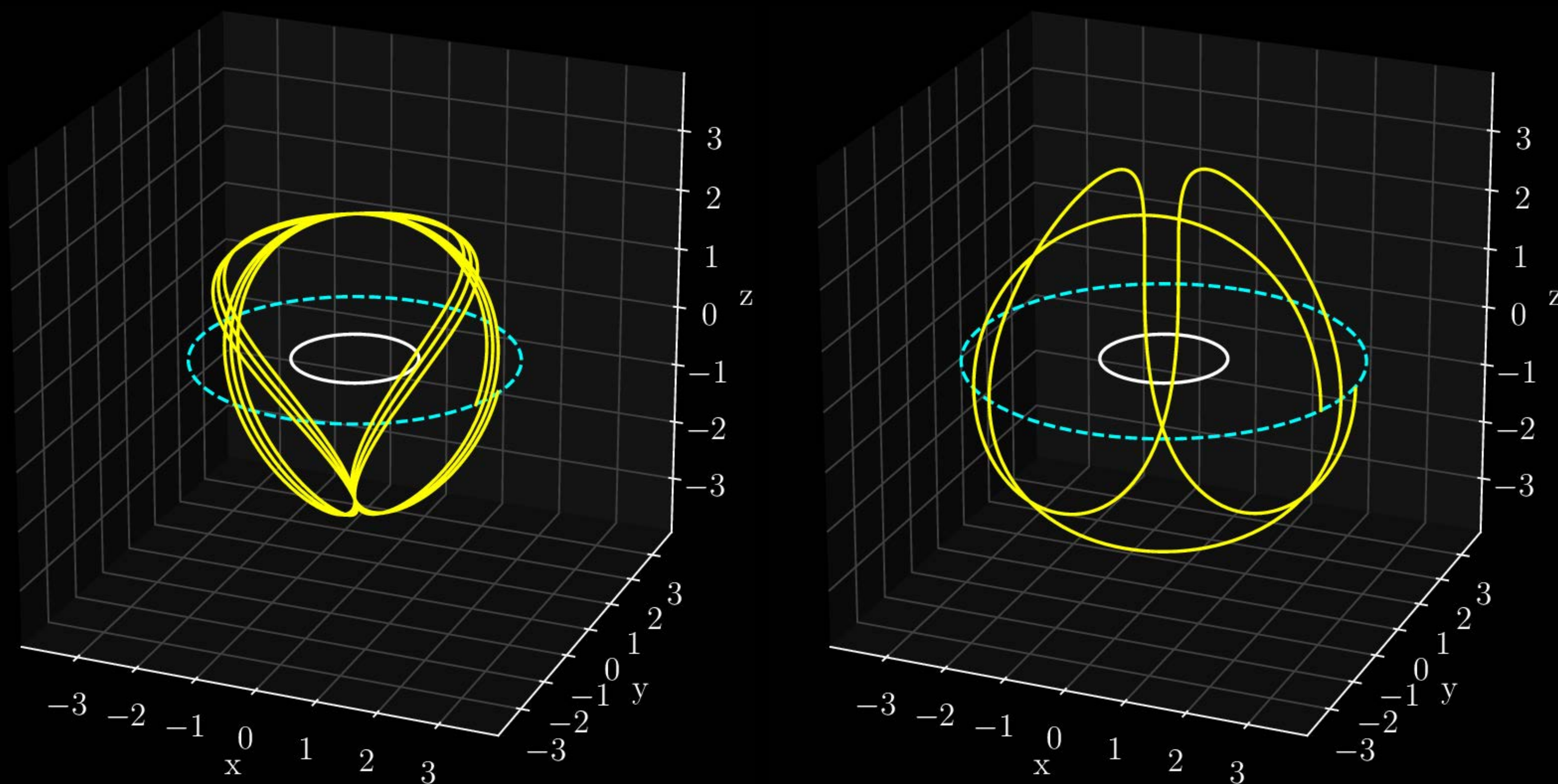
Pick out a "column"   Pick out a "row"

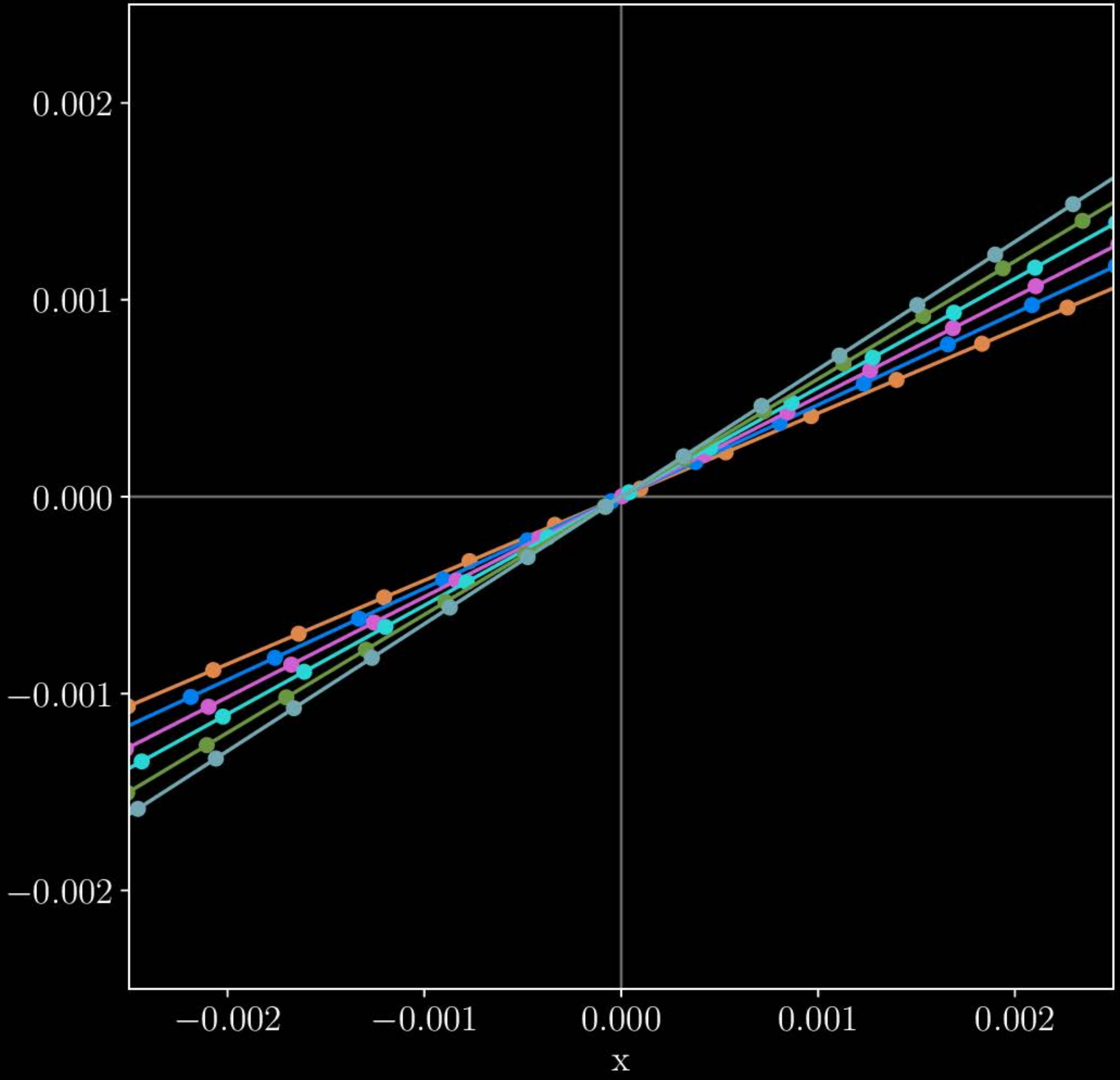
$$\frac{du^\mu}{d\lambda} = \left[-\left(\eta^{\mu\beta}u^\alpha - \frac{1}{2}\eta^{\mu\alpha}u^\beta\right) + f l^\mu\left(l^\beta u^\alpha - \frac{1}{2}l^\alpha u^\beta\right)\right]\partial_\alpha(f l_\beta l_\gamma u^\gamma)$$

Compute a scalar once for all μ

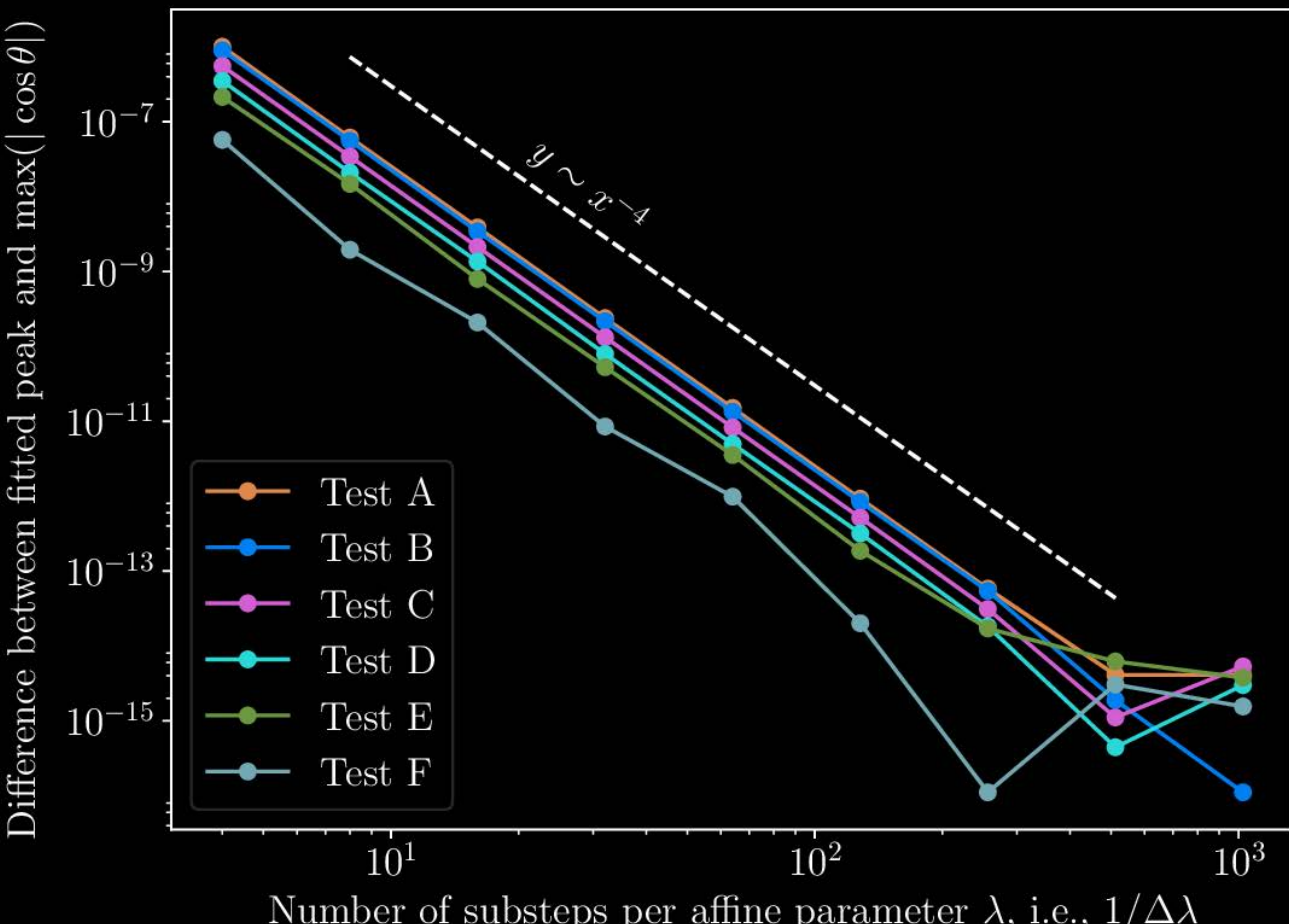
- Many operations can be sped up (reorder, fma, dot-product)
- Floating-point operation count: 190 flop vs 104 flop (132 vs 80 w/fma)

Convergence Test: Teo’s Unstable Spherical Photon Orbits



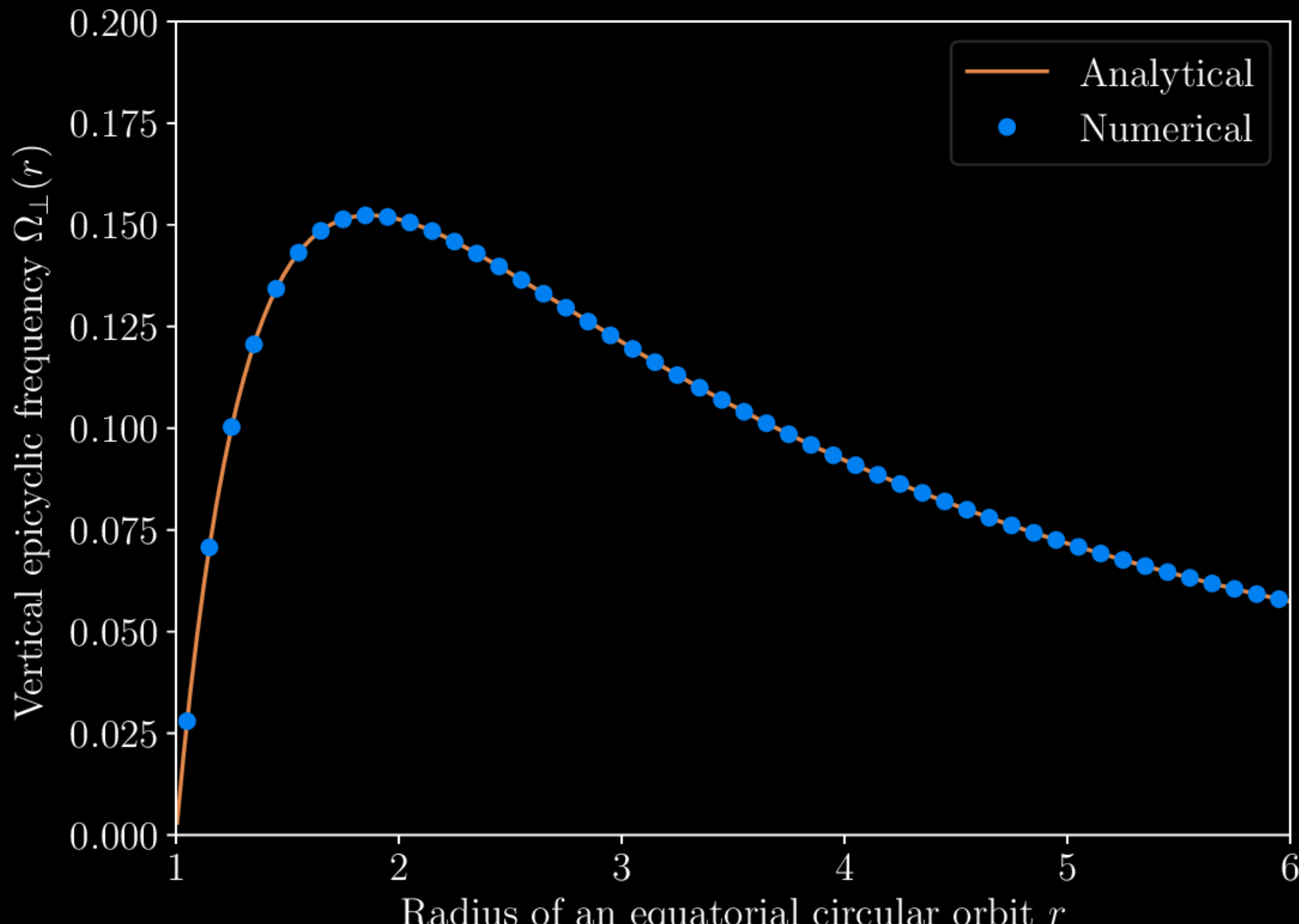


0.002
0.001
0.000
−0.001
−0.002
y
−0.002
−0.001
0.000
0.001
0.002
x



Difference between fitted peak and max(|cos θ|)
10^{-7}
10^{-9}
10^{-11}
10^{-13}
10^{-15}
$y \sim x^{-4}$
Test A
Test B
Test C
Test D
Test E
Test F
10^{1}
10^{2}
10^{3}
Number of substeps per affine parameter λ, i.e., 1/Δλ

Convergence Test: Oscillations of Stable Circular Particle Orbits



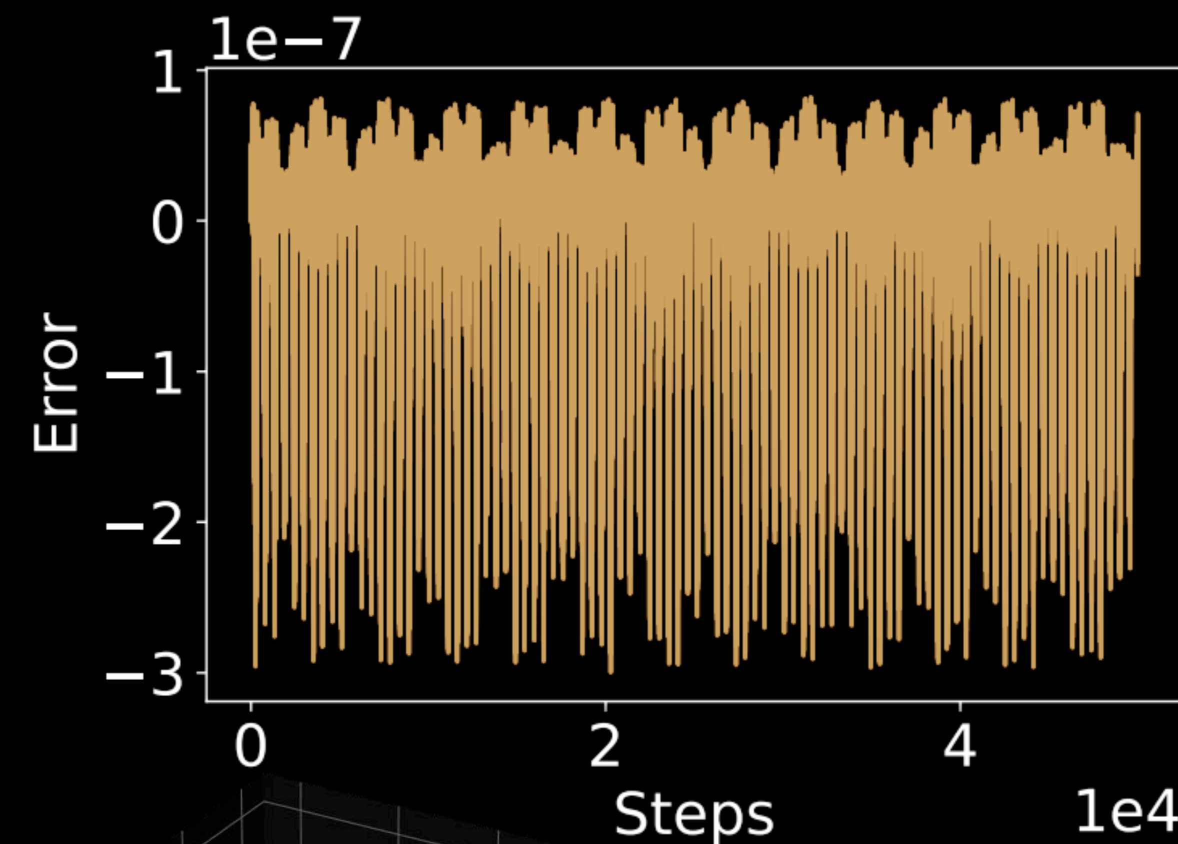
GRay2 Benchmark

Unit: ns	i7-3720QM	E5-2650x2	GT650M	K20X	GTX780	Titan Black
BL+AoS	55.4	16.6	70.6	3.90	6.44	1.65
BL+SoA	56.7	16.2	80.6	3.90	6.44	1.66
KS+AoS	66.0	17.4	59.1	2.41	6.95	1.15
KS+SoA	63.1	17.5	59.0	2.40	6.96	1.15

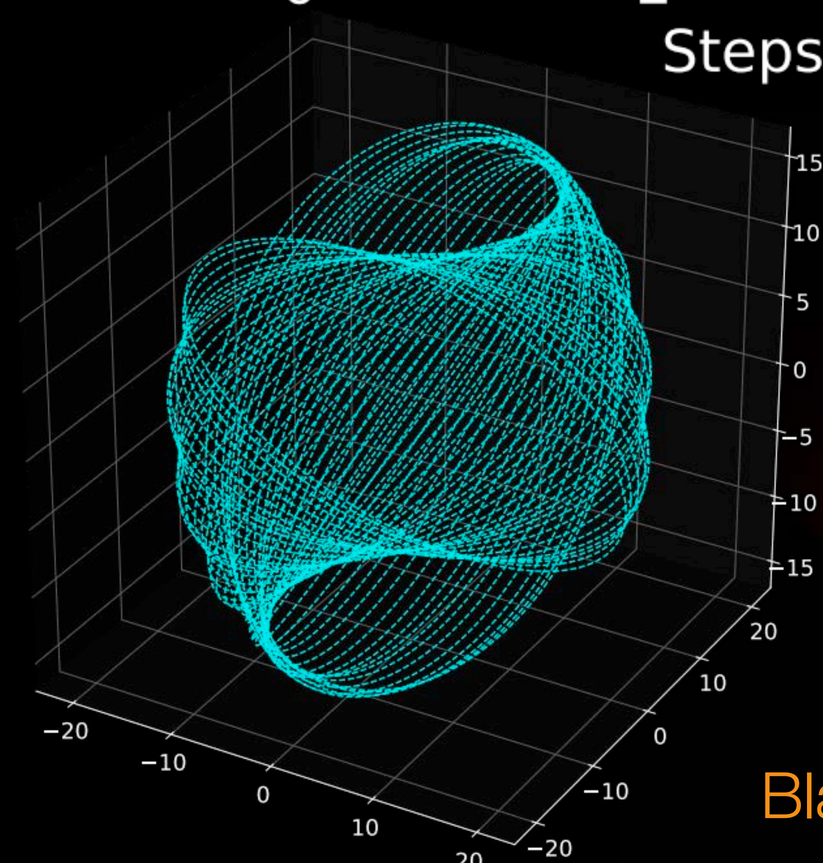
- GRay2 runs on CPU, integrated GPUs, discrete GPUs, & Xeon Phi
- Kerr-Schild on GPUs is super fast!!!

FANTASY: Symplectic + Autodiff

- Symplectic integrators based on Pihajoki (2015) and Tao (2016)
- Bounded error—ideal for long term evolutions of particles
- Self-contained implementation of autodiff in python
- The only required input is the metric—all the metric derivatives are computed automatically



Christian & Chan (2020)



Polarized Radiative Transfer

- Dexter (2016) `grtrans`
- Mościbrodzka & Gammie (2017) `ipole`
- Mościbrodzka (2020) `radpol`
- Bronzwaer et al. (2020) `raptor2`
- Prather et al. EHT polarized radiative transfer compression project

Scattering in Radiative Transfer

- Necessary for inverse Compton
- Monte Carlo approach:
  - Dolence et al. (2009) igrmonty
  - Davelaar et al. (2020) kmonty
- Short characteristics:
  - Narayan et al. (2016) HEROIC
- Long characteristics?

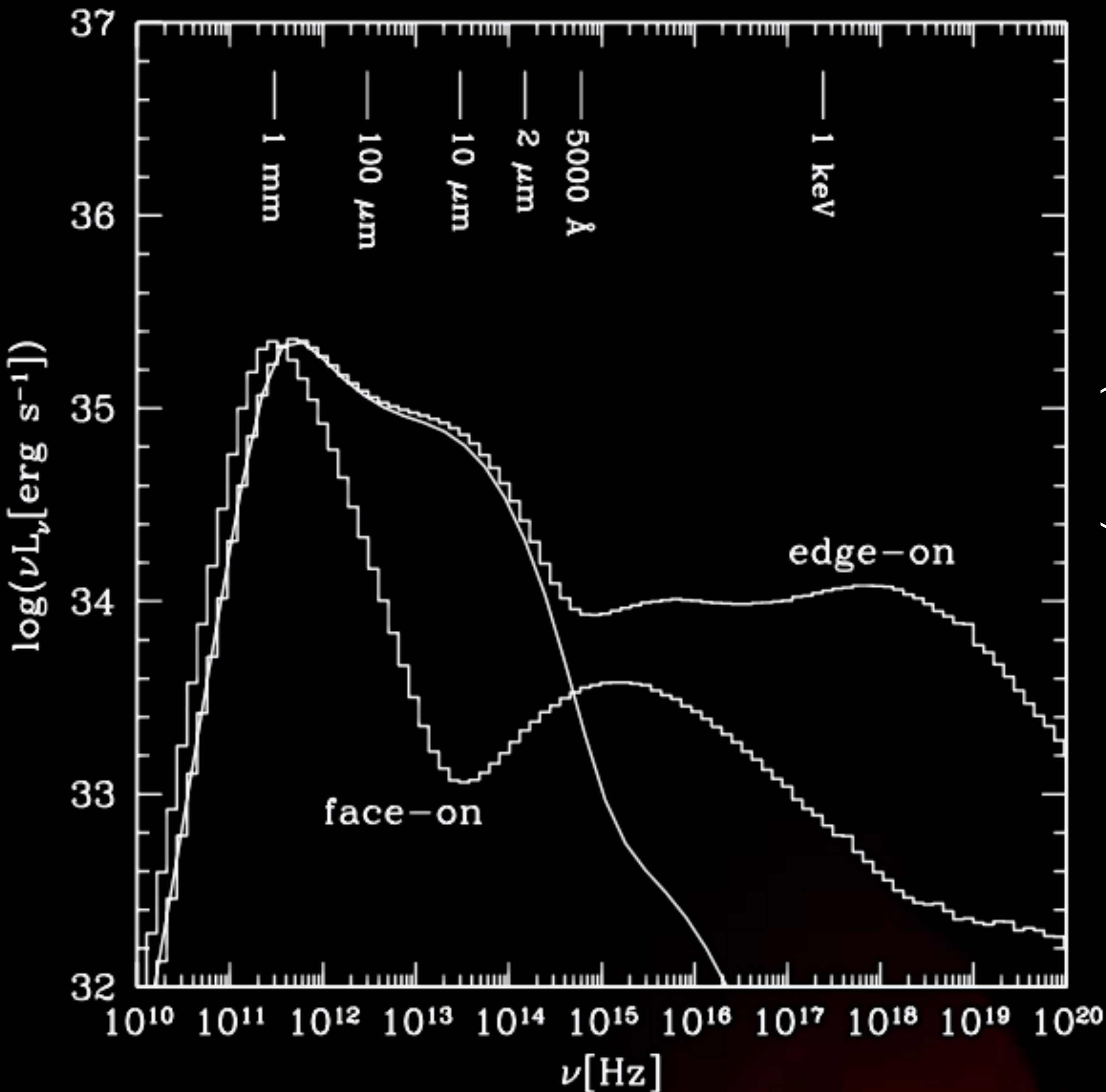


Figure 16. Same as Figure 14 except Compton scattering is included. The histograms show the grmonty result for nearly edge-on and face-on inclinations and the solid line is the ibothros spectrum for a nearly edge-on inclination.

Dolence et al. (2009)

Current Work

Charged Particles

Trent, Özel, Psaltis, et al.

Dynamic Spacetime

Bozzola, Paschalidis, et al.

Summary

- General relativistic ray tracing (GRRT) interfaces black hole theory and observations
- More important than ever because of event horizon scale resolution images thanks to the EHT!
- Two main concepts: geodesic integration, radiative transfer
- Numerical methods well established; foundation of many related numerical techniques
- Connect gravity with astrophysics and plasma physics
- Great time to work on black holes!

Ray Tracing Webinar Survey

http://bit.ly/RayTracingSurvey